Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.13-d+e-x-ˆm-sin-a+b-x+c-xˆ2-ˆn

Nasser M. Abbasi

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from https://rulebasedintegration.org

The number of integrals in this report is [36]. This is test number [71].

1.1   Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.

2. Rubi 4.16.1 in Mathematica 12 on windows 10.

3. Maple 2020 (64 bit) on windows 10.

4. Maxima 5.43 on Linux. (via sagemath 8.9)

5. Fricas 1.3.6 on Linux (via sagemath 9.0)

6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.

7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (36)	% 0. (0)
Mathematica	% 100. (36)	% 0. (0)
Maple	% 94.44 (34)	% 5.56 (2)
Maxima	% 94.44 (34)	% 5.56 (2)
Fricas	% 100. (36)	% 0. (0)
Sympy	% 55.56 (20)	% 44.44 (16)
Giac	% 94.44 (34)	% 5.56 (2)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	100.	0.	0.	0.
Maple	94.44	0.	0.	5.56
Maxima	27.78	0.	66.67	5.56
Fricas	100.	0.	0.	0.
Sympy	50.	5.56	0.	44.44
Giac	27.78	0.	66.67	5.56

The following is a Bar chart illustration of the data in the above table.
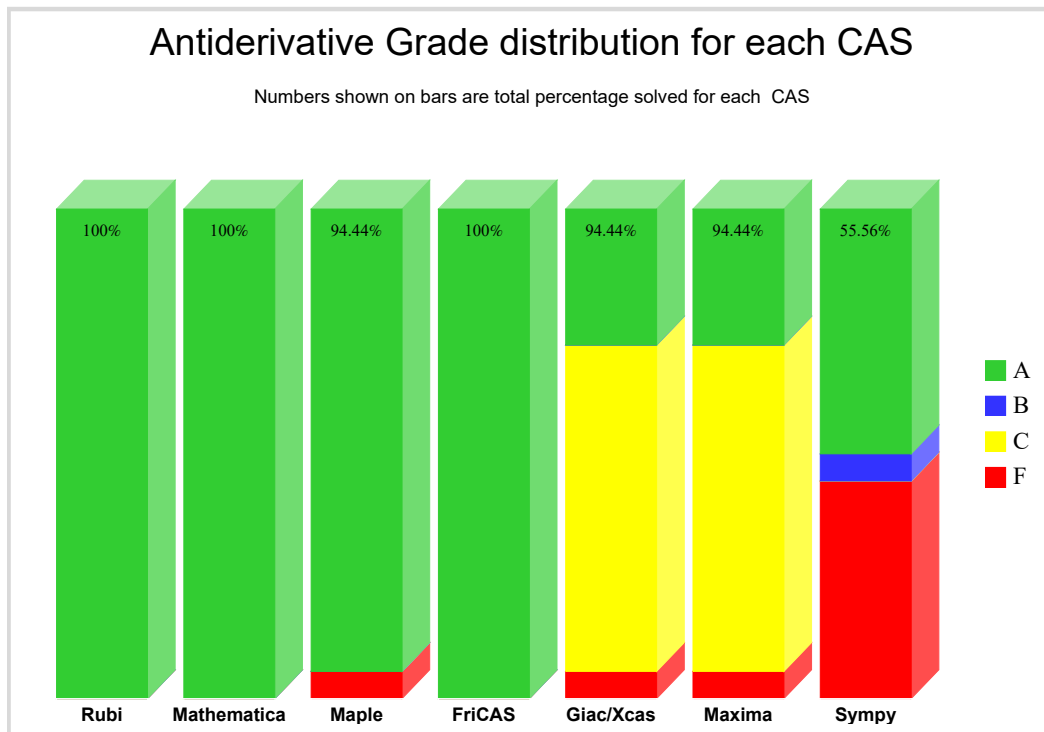
The figure below compares the CAS systems for each grade level.
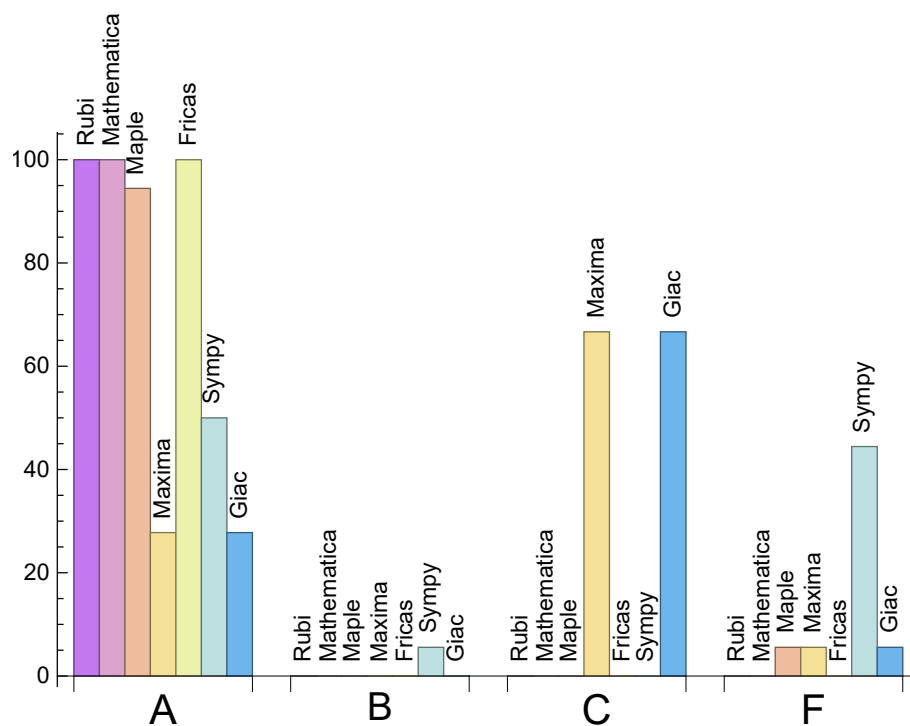
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	96.61	0.72	97.5	1.
Mathematica	2.69	79.36	0.64	86.5	0.87
Maple	0.04	88.59	0.61	72.	0.76
Maxima	1.92	1198.62	6.43	393.	4.05
Fricas	1.13	221.58	1.82	238.	2.38
Sympy	0.47	42.8	0.69	11.	0.41
Giac	1.08	175.94	1.28	165.	1.44

1.4 list of integrals that has no closed form antiderivative

$\{4, 9, 14, 15, 19, 23, 27, 28, 32, 36\}$

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7   Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A <u>time limit</u> of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8   Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9   Important notes about some of the results

1.9.1   Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10   Design of the test system

The following diagram gives a high level view of the current test build system.

High level overview of the CAS independent integration test build system
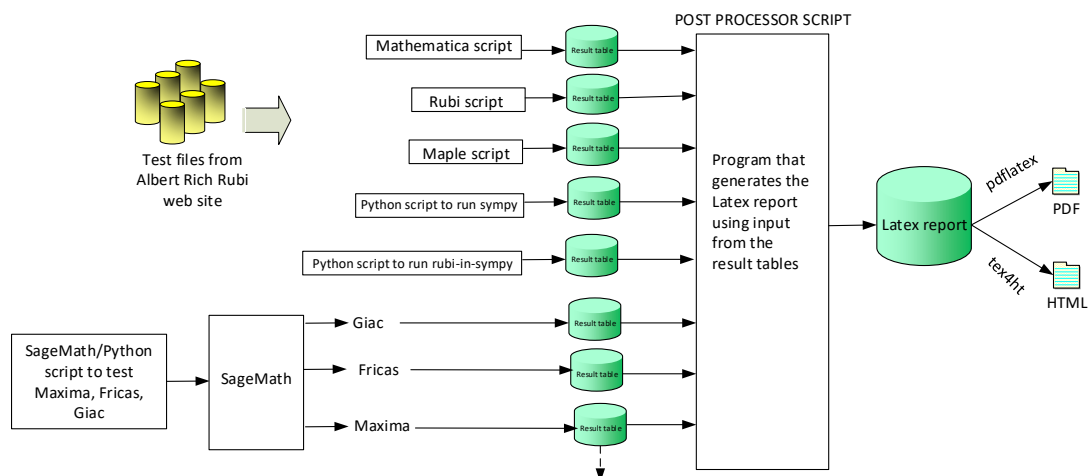
One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

Nasser M. Abbasi
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 }

B grade: { }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 }

B grade: { }

C grade: { }

F grade: { 5, 10 }

2.1.4   Maxima

A grade: { 4, 9, 14, 15, 19, 23, 27, 28, 32, 36

B grade: { }

C grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 16, 17, 18, 20, 21, 22, 24, 25, 26, 29, 30, 31, 33, 34, 35 }

F grade: { 5, 10 }

2.1.5   FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36 }

B grade: { }

C grade: { }

F grade: { }

2.1.6   Sympy

A grade: { 3, 4, 8, 9, 13, 14, 15, 18, 19, 22, 23, 26, 27, 28, 31, 32, 35, 36 }

B grade: { 12, 25 }

C grade: { }

F grade: { 1, 2, 5, 6, 7, 10, 11, 16, 17, 20, 21, 24, 29, 30, 33, 34 }

2.1.7   Giac

A grade: { 4, 9, 14, 15, 19, 23, 27, 28, 32, 36 }

B grade: { }

C grade: { 1, 2, 3, 6, 7, 8, 11, 12, 13, 16, 17, 18, 20, 21, 22, 24, 25, 26, 29, 30, 31, 33, 34, 35 }

F grade: { 5, 10 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	157	204	4115	448	0	306
normalized size	1	1.	0.63	0.82	16.53	1.8	0.	1.23
time (sec)	N/A	0.243	0.707	0.009	5.036	1.555	0.	2.972

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	111	100	1305	325	0	247
normalized size	1	1.	0.9	0.81	10.61	2.64	0.	2.01
time (sec)	N/A	0.037	0.371	0.007	2.729	1.562	0.	1.221

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	84	82	393	279	88	182
normalized size	1	1.	0.87	0.85	4.05	2.88	0.91	1.88
time (sec)	N/A	0.021	0.173	0.007	2.316	1.35	0.547	1.215

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	4.173	0.047	0.	0.	0.	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	110	0	0	311	0	0
normalized size	1	1.	1.	0.	0.	2.83	0.	0.
time (sec)	N/A	0.083	0.793	0.198	0.	1.413	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	165	202	4122	443	0	309
normalized size	1	1.	0.66	0.8	16.42	1.76	0.	1.23
time (sec)	N/A	0.204	0.712	0.01	2.285	1.607	0.	1.738

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	117	99	1310	323	0	250
normalized size	1	1.	0.94	0.8	10.56	2.6	0.	2.02
time (sec)	N/A	0.044	0.318	0.008	1.708	1.529	0.	2.446

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	89	80	393	278	94	185
normalized size	1	1.	0.91	0.82	4.01	2.84	0.96	1.89
time (sec)	N/A	0.025	0.185	0.009	2.049	1.585	0.545	2.795

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	4.792	0.046	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	115	0	0	309	0	0
normalized size	1	1.	1.05	0.	0.	2.81	0.	0.
time (sec)	N/A	0.084	0.802	0.204	0.	1.583	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	66	59	211	228	0	101
normalized size	1	1.	0.8	0.72	2.57	2.78	0.	1.23
time (sec)	N/A	0.033	0.117	0.009	3.979	1.483	0.	2.418

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	39	30	166	124	160	88
normalized size	1	1.	0.95	0.73	4.05	3.02	3.9	2.15
time (sec)	N/A	0.013	0.05	0.009	4.143	1.39	1.148	1.297

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	95	89	29	53
normalized size	1	1.	1.	0.83	3.96	3.71	1.21	2.21
time (sec)	N/A	0.006	0.027	0.007	3.77	1.354	0.5	1.175

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	14.481	0.063	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	52	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	11.554	0.066	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	170	191	4236	440	0	289
normalized size	1	1.	0.69	0.77	17.08	1.77	0.	1.17
time (sec)	N/A	0.227	0.655	0.018	3.616	1.609	0.	1.243

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	118	95	1372	315	0	230
normalized size	1	1.	0.94	0.75	10.89	2.5	0.	1.83
time (sec)	N/A	0.073	0.28	0.016	2.12	1.633	0.	1.248

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	72	410	238	83	165
normalized size	1	1.	0.97	0.72	4.1	2.38	0.83	1.65
time (sec)	N/A	0.05	0.103	0.013	1.888	1.535	1.114	1.278

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	6.992	0.099	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	175	199	4238	435	0	292
normalized size	1	1.	0.71	0.8	17.09	1.75	0.	1.18
time (sec)	N/A	0.22	0.73	0.017	2.208	1.497	0.	1.287

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	122	99	1372	313	0	232
normalized size	1	1.	0.97	0.79	10.89	2.48	0.	1.84
time (sec)	N/A	0.066	0.291	0.014	1.952	1.437	0.	1.403

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	102	76	410	235	88	167
normalized size	1	1.	1.02	0.76	4.1	2.35	0.88	1.67
time (sec)	N/A	0.048	0.11	0.015	2.073	1.536	1.145	1.346

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	7.299	0.097	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	64	231	215	0	86
normalized size	1	1.	0.91	0.75	2.72	2.53	0.	1.01
time (sec)	N/A	0.062	0.145	0.017	2.33	1.653	0.	1.352

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	42	35	182	135	121	73
normalized size	1	1.	0.91	0.76	3.96	2.93	2.63	1.59
time (sec)	N/A	0.03	0.061	0.014	2.042	1.66	1.648	1.289

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	46	74	22	35
normalized size	1	1.	1.	0.74	1.7	2.74	0.81	1.3
time (sec)	N/A	0.015	0.013	0.013	1.737	1.685	0.887	1.281

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	10.992	0.106	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	65	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	11.036	0.162	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	186	399	5825	567	0	768
normalized size	1	1.	0.65	1.4	20.44	1.99	0.	2.69
time (sec)	N/A	0.271	1.3	0.01	3.952	1.797	0.	1.261

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	128	184	1702	359	0	441
normalized size	1	1.	0.91	1.31	12.16	2.56	0.	3.15
time (sec)	N/A	0.056	0.668	0.01	2.714	1.741	0.	1.282

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	84	82	393	279	88	182
normalized size	1	1.	0.87	0.85	4.05	2.88	0.91	1.88
time (sec)	N/A	0.022	0.095	0.007	2.047	1.78	0.58	1.216

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.011	7.652	0.115	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	215	378	6030	608	0	726
normalized size	1	1.	0.74	1.3	20.72	2.09	0.	2.49
time (sec)	N/A	0.337	1.19	0.017	3.79	1.71	0.	1.349

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	140	170	1786	369	0	410
normalized size	1	1.	0.93	1.13	11.91	2.46	0.	2.73
time (sec)	N/A	0.093	0.437	0.016	2.678	1.539	0.	1.25

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	97	72	410	238	83	165
normalized size	1	1.	0.97	0.72	4.1	2.38	0.83	1.65
time (sec)	N/A	0.048	0.073	0.013	2.167	1.467	1.2	1.322

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	43	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	7.409	0.23	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [24] had the largest ratio of [0.4667]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.	15	0.4
2	A	4	4	1.	13	0.308
3	A	3	3	1.	11	0.273
4	A	0	0	0.	0	0.
5	A	5	4	1.	33	0.121
6	A	8	6	1.	16	0.375
7	A	4	4	1.	14	0.286
8	A	3	3	1.	12	0.25
Continued on next page						

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	0	0	0.	0	0.
10	A	5	4	1.	35	0.114
11	A	6	6	1.	13	0.462
12	A	3	3	1.	11	0.273
13	A	2	2	1.	9	0.222
14	A	0	0	0.	0	0.
15	A	0	0	0.	0	0.
16	A	10	7	1.	17	0.412
17	A	6	5	1.	15	0.333
18	A	5	4	1.	13	0.308
19	A	0	0	0.	0	0.
20	A	10	7	1.	18	0.389
21	A	6	5	1.	16	0.312
22	A	5	4	1.	14	0.286
23	A	0	0	0.	0	0.
24	A	8	7	1.	15	0.467
25	A	5	4	1.	13	0.308
26	A	4	3	1.	11	0.273
27	A	0	0	0.	0	0.
28	A	0	0	0.	0	0.
29	A	8	6	1.	19	0.316
30	A	4	4	1.	17	0.235
31	A	3	3	1.	11	0.273
32	A	0	0	0.	0	0.
33	A	10	7	1.	21	0.333
34	A	6	5	1.	19	0.263
35	A	5	4	1.	13	0.308
36	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^2 \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=249

$$\frac{\sqrt{\frac{\pi}{2}}b^2 \sin\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sin\left(a - \frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}b^2 \cos}{}$$

```
[Out] (b*Cos[a + b*x + c*x^2])/(4*c^2) - (x*Cos[a + b*x + c*x^2])/(2*c) + (Sqrt[P
i/2]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3
/2)) + (b^2*Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqr
t[2*Pi])])/(4*c^(5/2)) + (b^2*Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt
[2*Pi])]*Sin[a - b^2/(4*c)])/(4*c^(5/2)) - (Sqrt[Pi/2]*FresnelS[(b + 2*c*x)
/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])/(2*c^(3/2))
```

Rubi [A]   time = 0.242831, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = $ 0.4, Rules used = {3463, 3448, 3352, 3351, 3461, 3447}

$$\frac{\sqrt{\frac{\pi}{2}}b^2 \sin\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sin\left(a - \frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}b^2 \cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] (b*Cos[a + b*x + c*x^2])/(4*c^2) - (x*Cos[a + b*x + c*x^2])/(2*c) + (Sqrt[P
i/2]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3
/2)) + (b^2*Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqr
t[2*Pi])])/(4*c^(5/2)) + (b^2*Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt
[2*Pi])]*Sin[a - b^2/(4*c)])/(4*c^(5/2)) - (Sqrt[Pi/2]*FresnelS[(b + 2*c*x)
/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])/(2*c^(3/2))
```

Rule 3463

```
Int[((d_.) + (e_.)*(x_))^(m_)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> -Simp[(e*(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2])/(2*c), x] + (Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Cos[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
```

```
b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2 \sin\left(a + bx + cx^2\right)\, dx = -\frac{x\cos\left(a + bx + cx^2\right)}{2c} + \frac{\int \cos\left(a + bx + cx^2\right)\, dx}{2c} - \frac{b\int x\sin\left(a + bx + cx^2\right)\, dx}{2c}$$

$$= \frac{b\cos\left(a + bx + cx^2\right)}{4c^2} - \frac{x\cos\left(a + bx + cx^2\right)}{2c} + \frac{b^2 \int \sin\left(a + bx + cx^2\right)\, dx}{4c^2} + \frac{\cos\left(a - \frac{b^2}{4c}\right)}{}$$

$$= \frac{b\cos\left(a + bx + cx^2\right)}{4c^2} - \frac{x\cos\left(a + bx + cx^2\right)}{2c} + \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)C\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c}$$

$$= \frac{b\cos\left(a + bx + cx^2\right)}{4c^2} - \frac{x\cos\left(a + bx + cx^2\right)}{2c} + \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)C\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{b^2\sqrt{\frac{\pi}{2}}\cos\left(a\right)}{}$$

Mathematica [A]   time = 0.707047, size = 157, normalized size = 0.63

$$\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)\left(b^2\sin\left(a - \frac{b^2}{4c}\right) + 2c\cos\left(a - \frac{b^2}{4c}\right)\right) + \sqrt{2\pi}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)\left(b^2\cos\left(a - \frac{b^2}{4c}\right) - 2c\sin\left(a - \frac{b^2}{4c}\right)\right) + 2\sqrt{c}(b}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*(b - 2*c*x)*Cos[a + x*(b + c*x)] + Sqrt[2*Pi]*FresnelC[(b + 2*c*
x)/(Sqrt[c]*Sqrt[2*Pi])]*(2*c*Cos[a - b^2/(4*c)] + b^2*Sin[a - b^2/(4*c)])
+ Sqrt[2*Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*(b^2*Cos[a - b^2/(4
*c)] - 2*c*Sin[a - b^2/(4*c)])))/(8*c^(5/2))
```

Maple [A]   time = 0.009, size = 204, normalized size = 0.8

$$-\frac{x\cos\left(cx^2 + bx + a\right)}{2c} - \frac{b}{2c}\left(-\frac{\cos\left(cx^2 + bx + a\right)}{2c} - \frac{b\sqrt{2}\sqrt{\pi}}{4}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right)\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c*x^2+b*x+a),x)
```

[Out] -1/2*x*cos(c*x^2+b*x+a)/c-1/2*b/c*(-1/2*cos(c*x^2+b*x+a)/c-1/4*b/c^(3/2)*2^
(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x
+1/2*b))-sin((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b)
)))+1/4/c^(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^
(1/2)/c^(1/2)*(c*x+1/2*b))+sin((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c
^(1/2)*(c*x+1/2*b)))

<u>Maxima</u> [C]   time = 5.03571, size = 4115, normalized size = 16.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(c*x^2+b*x+a),x, algorithm="maxima")

[Out] 1/16*((4*b*c*(e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + e^(-1/4*(4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) + b*c*(4*I*
e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 4*I*e^(-1/4*(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*((4*c^2*x^2 + 4*b*c*
x + b^2)/abs(c))^(3/2) + (b^3*(-4*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x
 + I*b^2)/c) + 4*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*ab
s(c)*cos(-1/4*(b^2 - 4*a*c)/c) - 4*b^3*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b
*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*ab
s(c)*sin(-1/4*(b^2 - 4*a*c)/c) + (c^3*(-32*I*gamma(3/2, 1/4*(4*I*c^2*x^2 +
4*I*b*c*x + I*b^2)/c) + 32*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b
^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - 32*c^3*(gamma(3/2, 1/4*(4*I*c^2*
x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*
b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*x^3 + (b*c^2*(-48*I*gamma(3/2, 1
/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + 48*I*gamma(3/2, -1/4*(4*I*c^2*x^2
 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - 48*b*c^2*(gamm
a(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*
x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*x^2 + (b^2*c
*(-24*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + 24*I*gamma(3/
2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)
/c) - 24*b^2*c*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma
(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a
*c)/c))*x)*cos(3/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + ((I*sqrt(pi
)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) - I*sqrt(pi)*(er
f(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(-1/4*(b^2 -
 4*a*c)/c) + (sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
- 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*
b^5*sin(-1/4*(b^2 - 4*a*c)/c) + ((8*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 +

```
4*I*b*c*x + I*b^2)/c)) - 1) - 8*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/4*(b^2 - 4*a*c)/c) + 8*(sqrt(pi)
*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1
/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*sin(-1/4*(b^2
- 4*a*c)/c))*x^3 + ((12*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x +
 I*b^2)/c)) - 1) - 12*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x +
I*b^2)/c)) - 1))*b^3*c^2*cos(-1/4*(b^2 - 4*a*c)/c) + 12*(sqrt(pi)*(erf(1/2*
sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(
4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(-1/4*(b^2 - 4*a*c)/c
))*x^2 + ((6*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
 - 1) - 6*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) -
 1))*b^4*c*cos(-1/4*(b^2 - 4*a*c)/c) + 6*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x
^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/4*(b^2 - 4*a*c)/c))*x)*cos(1/2*arc
tan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) - (4*b^3*(gamma(3/2, 1/4*(4*I*c^2*x
^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b
^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - b^3*(4*I*gamma(3/2, 1/4*(4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c) - 4*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x
 + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c) + (32*c^3*(gamma(3/2, 1/4*(4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c
*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - c^3*(32*I*gamma(3/2, 1/4
*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 32*I*gamma(3/2, -1/4*(4*I*c^2*x^2 +
 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*x^3 + (48*b*c^2*(
gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*
c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - b*c^2*(
48*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 48*I*gamma(3/2,
-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c)
)*x^2 + (24*b^2*c*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + ga
mma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 -
4*a*c)/c) - b^2*c*(24*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)
 - 24*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1
/4*(b^2 - 4*a*c)/c))*x)*sin(3/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0))
+ ((sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqr
t(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(-1
/4*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x +
 I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b
^2)/c)) - 1))*b^5*sin(-1/4*(b^2 - 4*a*c)/c) + (8*(sqrt(pi)*(erf(1/2*sqrt((4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/4*(b^2 - 4*a*c)/c) + (-8
*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 8*I*
sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3
*sin(-1/4*(b^2 - 4*a*c)/c))*x^3 + (12*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*
b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/4*(b^2 - 4*a*c)/c) + (-12*I*sqrt(pi
)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 12*I*sqrt(pi)*
```

```
(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(-1/4
*(b^2 - 4*a*c)/c))*x^2 + (6*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*
x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*
b^2)/c)) - 1))*b^4*c*cos(-1/4*(b^2 - 4*a*c)/c) + (-6*I*sqrt(pi)*(erf(1/2*sq
rt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 6*I*sqrt(pi)*(erf(1/2*sqrt(
-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/4*(b^2 - 4*a*c)/c
))*x)*sin(1/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)))/(c^3*((4*c^2*x^2
+ 4*b*c*x + b^2)/abs(c))^(3/2)*abs(c))
```

Fricas [A]   time = 1.55497, size = 448, normalized size = 1.8

$$\frac{\sqrt{2}\left(\pi b^2 \sin\left(-\frac{b^2-4\,ac}{4\,c}\right) + 2\,\pi c \cos\left(-\frac{b^2-4\,ac}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right) + \sqrt{2}\left(\pi b^2 \cos\left(-\frac{b^2-4\,ac}{4\,c}\right) - 2\,\pi c \sin\left(-\frac{b^2-4\,ac}{4\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}}{2}\right.}{8\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(pi*b^2*sin(-1/4*(b^2 - 4*a*c)/c) + 2*pi*c*cos(-1/4*(b^2 - 4*a
*c)/c))*sqrt(c/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x + b)*sqrt(c/pi)/c) + sqrt
(2)*(pi*b^2*cos(-1/4*(b^2 - 4*a*c)/c) - 2*pi*c*sin(-1/4*(b^2 - 4*a*c)/c))*s
qrt(c/pi)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b)*sqrt(c/pi)/c) - 2*(2*c^2*x -
b*c)*cos(c*x^2 + b*x + a))/c^3
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin\left(a + bx + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(x**2*sin(a + b*x + c*x**2), x)
```

Giac [C]   time = 2.97246, size = 306, normalized size = 1.23

$$-\frac{-\dfrac{i\sqrt{2}\sqrt{\pi}\left(b^2+2ic\right)\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac}{4c}\right)}}{\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}-2i\left(c\left(2ix+\frac{ib}{c}\right)-2ib\right)e^{\left(icx^2+ibx+ia\right)}}{16\,c^2}-\frac{\dfrac{i\sqrt{2}\sqrt{\pi}\left(b^2-2ic\right)\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\right.}{\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(-I*sqrt(2)*sqrt(pi)*(b^2 + 2*I*c)*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c
/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c)/c)/((-I*c/abs(c) + 1)*
sqrt(abs(c))) - 2*I*(c*(2*I*x + I*b/c) - 2*I*b)*e^(I*c*x^2 + I*b*x + I*a))/
c^2 - 1/16*(I*sqrt(2)*sqrt(pi)*(b^2 - 2*I*c)*erf(-1/4*sqrt(2)*(2*x + b/c)*(
I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c)/c)/((I*c/abs(c) +
1)*sqrt(abs(c))) - 2*I*(c*(2*I*x + I*b/c) - 2*I*b)*e^(-I*c*x^2 - I*b*x - I*
a))/c^2
```

3.2 $\int x \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=123

$$-\frac{\sqrt{\frac{\pi}{2}}b\sin\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}b\cos\left(a - \frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{\cos\left(a + bx + cx^2\right)}{2c}$$

```
[Out] -Cos[a + b*x + c*x^2]/(2*c) - (b*Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b
+ 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) - (b*Sqrt[Pi/2]*FresnelC[(b + 2
*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])/(2*c^(3/2))
```

Rubi [A]   time = 0.0371549, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3461, 3447, 3351, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}b\sin\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}b\cos\left(a - \frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{\cos\left(a + bx + cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x + c*x^2],x]
```

```
[Out] -Cos[a + b*x + c*x^2]/(2*c) - (b*Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b
+ 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) - (b*Sqrt[Pi/2]*FresnelC[(b + 2
*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])/(2*c^(3/2))
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x \sin\left(a + bx + cx^2\right) dx = -\frac{\cos\left(a + bx + cx^2\right)}{2c} - \frac{b \int \sin\left(a + bx + cx^2\right) dx}{2c}$$

$$= -\frac{\cos\left(a + bx + cx^2\right)}{2c} - \frac{\left(b \cos\left(a - \frac{b^2}{4c}\right)\right) \int \sin\left(\frac{(b+2cx)^2}{4c}\right) dx}{2c} - \frac{\left(b \sin\left(a - \frac{b^2}{4c}\right)\right) \int \cos\left(\frac{(b+2cx)^2}{4c}\right)}{2c}$$

$$= -\frac{\cos\left(a + bx + cx^2\right)}{2c} - \frac{b\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{b\sqrt{\frac{\pi}{2}} C\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right) \sin\left(a - \frac{b^2}{4c}\right)}{2c^{3/2}}$$

Mathematica [A]   time = 0.371194, size = 111, normalized size = 0.9

$$-\frac{\sqrt{2\pi} b \sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right) + \sqrt{2\pi} b \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right) + 2\sqrt{c} \cos(a + x(b + cx))}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x + c*x^2],x]
```

```
[Out] -(2*Sqrt[c]*Cos[a + x*(b + c*x)] + b*Sqrt[2*Pi]*Cos[a - b^2/(4*c)]*FresnelS
[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])] + b*Sqrt[2*Pi]*FresnelC[(b + 2*c*x)/(Sqr
t[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])/(4*c^(3/2))
```

Maple [A]   time = 0.007, size = 100, normalized size = 0.8

$$-\frac{\cos\left(cx^2 + bx + a\right)}{2c} - \frac{b\sqrt{2}\sqrt{\pi}}{4}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right) \text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right) \text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c*x^2+b*x+a),x)
```

```
[Out] -1/2*cos(c*x^2+b*x+a)/c-1/4*b/c^(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*a)/c
)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b))-sin((1/4*b^2-c*a)/c)*Fresn
elC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b)))
```

Maxima [C]   time = 2.72878, size = 1305, normalized size = 10.61

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(((I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1)
 - I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^
2*cos(-1/4*(b^2 - 4*a*c)/c) + (sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x +
I*b^2)/c)) - 1))*b^2*sin(-1/4*(b^2 - 4*a*c)/c) + ((2*I*sqrt(pi)*(erf(1/2*sq
rt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) - 2*I*sqrt(pi)*(erf(1/2*sqrt(
-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*cos(-1/4*(b^2 - 4*a*c)/c)
+ 2*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sq
rt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-
1/4*(b^2 - 4*a*c)/c))*x)*cos(1/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0))
 + ((sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sq
rt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(-
1/4*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x
+ I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*
b^2)/c)) - 1))*b^2*sin(-1/4*(b^2 - 4*a*c)/c) + (2*(sqrt(pi)*(erf(1/2*sqrt((
4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*cos(-1/4*(b^2 - 4*a*c)/c) + (-2*I*
sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqr
t(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1
/4*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0))
+ (2*c*(e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + e^(-1/4*(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c))*cos(-1/4*(b^2 - 4*a*c)/c) + c*(2*I*e^(1/4*(4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c) - 2*I*e^(-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2
)/c))*sin(-1/4*(b^2 - 4*a*c)/c))*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c)))/
(c^2*sqrt((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c)))
```

Fricas [A]   time = 1.5618, size = 325, normalized size = 2.64

$$-\frac{\sqrt{2}\pi b\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{4\,c}\right)\mathrm{S}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)+\sqrt{2}\pi b\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)\sin\left(-\frac{b^2-4\,ac}{4\,c}\right)+2\,c\cos\left(cx^2+bx+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*pi*b*sqrt(c/pi)*cos(-1/4*(b^2 - 4*a*c)/c)*fresnel_sin(1/2*sqr
t(2)*(2*c*x + b)*sqrt(c/pi)/c) + sqrt(2)*pi*b*sqrt(c/pi)*fresnel_cos(1/2*sq
rt(2)*(2*c*x + b)*sqrt(c/pi)/c)*sin(-1/4*(b^2 - 4*a*c)/c) + 2*c*cos(c*x^2 +
 b*x + a))/c^2
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral(x*sin(a + b*x + c*x**2), x)
```

Giac [C]   time = 1.22087, size = 247, normalized size = 2.01

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac}{4c}\right)}}{\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}+2\,e^{\left(icx^2+ibx+ia\right)}}{8\,c}-\frac{-\frac{i\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac}{4c}\right)}}{\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}+2\,e^{\left(-icx^2-ib x\right.}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(2)*sqrt(pi)*b*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*s
qrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

```
+ 2*e^(I*c*x^2 + I*b*x + I*a))/c - 1/8*(-I*sqrt(2)*sqrt(pi)*b*erf(-1/4*sqrt
(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c)/c
)/((I*c/abs(c) + 1)*sqrt(abs(c))) + 2*e^(-I*c*x^2 - I*b*x - I*a))/c
```

3.3 $\int \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=97

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}}$$

```
[Out] (Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/
Sqrt[c] + (Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^
2/(4*c)])/Sqrt[c]
```

Rubi [A]   time = 0.0212291, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3447, 3351, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/
Sqrt[c] + (Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^
2/(4*c)])/Sqrt[c]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a + bx + cx^2\right)\, dx = \cos\left(a - \frac{b^2}{4c}\right) \int \sin\left(\frac{(b + 2cx)^2}{4c}\right)\, dx + \sin\left(a - \frac{b^2}{4c}\right) \int \cos\left(\frac{(b + 2cx)^2}{4c}\right)\, dx$$

$$= \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}} C\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)\sin\left(a - \frac{b^2}{4c}\right)}{\sqrt{c}}$$

Mathematica [A]   time = 0.172663, size = 84, normalized size = 0.87

$$\frac{\sqrt{\frac{\pi}{2}}\left(\sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right) + \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*(Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]
+ FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)]))/Sqrt[c]
```

Maple [A]   time = 0.007, size = 82, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{\pi}}{2}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right) \text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right) \text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right)\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*x^2+b*x+a),x)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/c^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2
)/c^(1/2)*(c*x+1/2*b))-sin((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/
2)*(c*x+1/2*b)))
```

Maxima [C]   time = 2.31587, size = 393, normalized size = 4.05

$$-\frac{\sqrt{\pi}\left(\left(\left(-i\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)-i\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)-\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*pi + 1/2*arctan2
(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) - (cos(1/4*pi + 1/2*arctan2(0, c)) + cos
(-1/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-
1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x +
I*b)/sqrt(I*c)) + ((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/
2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arct
an2(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) +
cos(-1/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*si
n(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x
 + I*b)/sqrt(-I*c)))/sqrt(abs(c))
```

Fricas [A]   time = 1.34997, size = 279, normalized size = 2.88

$$\frac{\sqrt{2}\pi\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{4c}\right)\mathrm{S}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2c}\right)+\sqrt{2}\pi\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2c}\right)\sin\left(-\frac{b^2-4\,ac}{4c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*pi*sqrt(c/pi)*cos(-1/4*(b^2 - 4*a*c)/c)*fresnel_sin(1/2*sqrt(2
)*(2*c*x + b)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/pi)*fresnel_cos(1/2*sqrt(2)
*(2*c*x + b)*sqrt(c/pi)/c)*sin(-1/4*(b^2 - 4*a*c)/c))/c
```

Sympy [A]   time = 0.547151, size = 88, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{\pi}\left(\sin\left(a-\frac{b^2}{4c}\right)\mathrm{C}\left(\frac{\sqrt{2}(b+2cx)}{2\sqrt{\pi}\sqrt{c}}\right)+\cos\left(a-\frac{b^2}{4c}\right)\mathrm{S}\left(\frac{\sqrt{2}(b+2cx)}{2\sqrt{\pi}\sqrt{c}}\right)\right)\sqrt{\frac{1}{c}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x**2+b*x+a),x)
```

```
[Out] sqrt(2)*sqrt(pi)*(sin(a - b**2/(4*c))*fresnelc(sqrt(2)*(b + 2*c*x)/(2*sqrt(
pi)*sqrt(c))) + cos(a - b**2/(4*c))*fresnels(sqrt(2)*(b + 2*c*x)/(2*sqrt(pi
)*sqrt(c))))*sqrt(1/c)/2
```

Giac [C]   time = 1.21517, size = 182, normalized size = 1.88

$$
\frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}} - \frac{i\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*sqrt(
abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) - 1/
4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt(abs
(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.4 $\int \dfrac{\sin\left(a+bx+cx^2\right)}{x}\,dx$

Optimal. Leaf size=17

$$\text{Unintegrable}\left(\frac{\sin\left(a + bx + cx^2\right)}{x}, x\right)$$

[Out] Unintegrable[Sin[a + b*x + c*x^2]/x, x]

Rubi [A]   time = 0.0082178, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin\left(a + bx + cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[a + b*x + c*x^2]/x,x]

[Out] Defer[Int][Sin[a + b*x + c*x^2]/x, x]

Rubi steps

$$\int \frac{\sin\left(a + bx + cx^2\right)}{x}\,dx = \int \frac{\sin\left(a + bx + cx^2\right)}{x}\,dx$$

Mathematica [A]   time = 4.17324, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + bx + cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + b*x + c*x^2]/x,x]

[Out] Integrate[Sin[a + b*x + c*x^2]/x, x]

Maple [A]   time = 0.047, size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c*x^2+b*x+a)/x,x)

[Out] int(sin(c*x^2+b*x+a)/x,x)

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(c*x^2+b*x+a)/x,x, algorithm="maxima")

[Out] integrate(sin(c*x^2 + b*x + a)/x, x)

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sin\left(cx^2 + bx + a\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(c*x^2+b*x+a)/x,x, algorithm="fricas")

[Out] integral(sin(c*x^2 + b*x + a)/x, x)

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + bx + cx^2\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(c*x**2+b*x+a)/x,x)
```

```
[Out] Integral(sin(a + b*x + c*x**2)/x, x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(c*x^2+b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(c*x^2 + b*x + a)/x, x)
```

3.5 $\int \left(-\dfrac{b\cos\left(a+bx+cx^2\right)}{x} + \dfrac{\sin\left(a+bx+cx^2\right)}{x^2} \right) dx$

Optimal. Leaf size=110

$$\sqrt{2\pi}\sqrt{c}\cos\left(a-\frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right) - \sqrt{2\pi}\sqrt{c}\sin\left(a-\frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right) - \frac{\sin\left(a+bx+cx^2\right)}{x}$$

```
[Out] Sqrt[c]*Sqrt[2*Pi]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*
Pi])] - Sqrt[c]*Sqrt[2*Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a
 - b^2/(4*c)] - Sin[a + b*x + c*x^2]/x
```

Rubi [A]   time = 0.0831945, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3465, 3448, 3352, 3351}

$$\sqrt{2\pi}\sqrt{c}\cos\left(a-\frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right) - \sqrt{2\pi}\sqrt{c}\sin\left(a-\frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right) - \frac{\sin\left(a+bx+cx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-((b*Cos[a + b*x + c*x^2])/x) + Sin[a + b*x + c*x^2]/x^2,x]
```

```
[Out] Sqrt[c]*Sqrt[2*Pi]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*
Pi])] - Sqrt[c]*Sqrt[2*Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a
 - b^2/(4*c)] - Sin[a + b*x + c*x^2]/x
```

Rule 3465

```
Int[((d_.) + (e_.)*(x_))^(m_)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Simp[((d + e*x)^(m + 1)*Sin[a + b*x + c*x^2])/(e*(m + 1)), x] + (-D
ist[(2*c)/(e^2*(m + 1)), Int[(d + e*x)^(m + 2)*Cos[a + b*x + c*x^2], x], x]
 - Dist[(b*e - 2*c*d)/(e^2*(m + 1)), Int[(d + e*x)^(m + 1)*Cos[a + b*x + c*
x^2], x], x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && LtQ[m,
 -1]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
```

```
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \left(-\frac{b\cos\left(a + bx + cx^2\right)}{x} + \frac{\sin\left(a + bx + cx^2\right)}{x^2} \right) dx = -\left(b\int \frac{\cos\left(a + bx + cx^2\right)}{x} dx \right) + \int \frac{\sin\left(a + bx + cx^2\right)}{x^2} dx$$

$$= -\frac{\sin\left(a + bx + cx^2\right)}{x} + (2c)\int \cos\left(a + bx + cx^2\right) dx$$

$$= -\frac{\sin\left(a + bx + cx^2\right)}{x} + \left(2c\cos\left(a - \frac{b^2}{4c}\right) \right) \int \cos\left(\frac{(b + 2cx)^2}{4c} \right) dx$$

$$= \sqrt{c}\sqrt{2\pi}\cos\left(a - \frac{b^2}{4c}\right) C\left(\frac{b + 2cx}{\sqrt{c}\sqrt{2\pi}} \right) - \sqrt{c}\sqrt{2\pi} S\left(\frac{b + 2cx}{\sqrt{c}\sqrt{2\pi}} \right) \sin\left(a -$$

Mathematica [A]   time = 0.792563, size = 110, normalized size = 1.

$$\sqrt{2\pi}\sqrt{c}\cos\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b + 2cx}{\sqrt{2\pi}\sqrt{c}} \right) - \frac{\sqrt{2\pi}\sqrt{c}x\sin\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}} \right) + \sin(a + x(b + cx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((b*Cos[a + b*x + c*x^2])/x) + Sin[a + b*x + c*x^2]/x^2,x]
```

```
[Out] Sqrt[c]*Sqrt[2*Pi]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*
Pi])] - (Sqrt[c]*Sqrt[2*Pi]*x*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Si
n[a - b^2/(4*c)] + Sin[a + x*(b + c*x)])/x
```

Maple [F]   time = 0.198, size = 0, normalized size = 0.

$$\int -\frac{b\cos\left(cx^2 + bx + a\right)}{x} + \frac{\sin\left(cx^2 + bx + a\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-b*cos(c*x^2+b*x+a)/x+sin(c*x^2+b*x+a)/x^2,x)

[Out] int(-b*cos(c*x^2+b*x+a)/x+sin(c*x^2+b*x+a)/x^2,x)

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\cos\left(cx^2 + bx + a\right)}{x} + \frac{\sin\left(cx^2 + bx + a\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-b*cos(c*x^2+b*x+a)/x+sin(c*x^2+b*x+a)/x^2,x, algorithm="maxima")

[Out] integrate(-b*cos(c*x^2 + b*x + a)/x + sin(c*x^2 + b*x + a)/x^2, x)

Fricas [A]   time = 1.41277, size = 311, normalized size = 2.83

$$\frac{\sqrt{2}\pi x\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{4\,c}\right)\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right) - \sqrt{2}\pi x\sqrt{\frac{c}{\pi}}\,\mathrm{S}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)\sin\left(-\frac{b^2-4\,ac}{4\,c}\right) - \sin\left(cx^2 + bx + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-b*cos(c*x^2+b*x+a)/x+sin(c*x^2+b*x+a)/x^2,x, algorithm="fricas")

[Out] (sqrt(2)*pi*x*sqrt(c/pi)*cos(-1/4*(b^2 - 4*a*c)/c)*fresnel_cos(1/2*sqrt(2)*
(2*c*x + b)*sqrt(c/pi)/c) - sqrt(2)*pi*x*sqrt(c/pi)*fresnel_sin(1/2*sqrt(2)
*(2*c*x + b)*sqrt(c/pi)/c)*sin(-1/4*(b^2 - 4*a*c)/c) - sin(c*x^2 + b*x + a)
)/x

49

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sin\left(a + bx + cx^2\right)}{x^2}\,dx - \int \frac{b\cos\left(a + bx + cx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*cos(c*x**2+b*x+a)/x+sin(c*x**2+b*x+a)/x**2,x)
```

```
[Out] -Integral(-sin(a + b*x + c*x**2)/x**2, x) - Integral(b*cos(a + b*x + c*x**2
)/x, x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\cos\left(cx^2 + bx + a\right)}{x} + \frac{\sin\left(cx^2 + bx + a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*cos(c*x^2+b*x+a)/x+sin(c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(-b*cos(c*x^2 + b*x + a)/x + sin(c*x^2 + b*x + a)/x^2, x)
```

3.6 $\int x^2 \sin\left(a + bx - cx^2\right) dx$

Optimal. Leaf size=251

$$-\frac{\sqrt{\frac{\pi}{2}}b^2 \sin\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sin\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \sqrt{\frac{\pi}{2}}b^2 \cos$$

[Out] (b*Cos[a + b*x - c*x^2])/(4*c^2) + (x*Cos[a + b*x - c*x^2])/(2*c) + (Sqrt[P
i/2]*Cos[a + b^2/(4*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3
/2)) + (b^2*Sqrt[Pi/2]*Cos[a + b^2/(4*c)]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqr
t[2*Pi])])/(4*c^(5/2)) - (b^2*Sqrt[Pi/2]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt
[2*Pi])]*Sin[a + b^2/(4*c)])/(4*c^(5/2)) + (Sqrt[Pi/2]*FresnelS[(b - 2*c*x)
/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^2/(4*c)])/(2*c^(3/2))

Rubi [A]   time = 0.203884, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3463, 3448, 3352, 3351, 3461, 3447}

$$-\frac{\sqrt{\frac{\pi}{2}}b^2 \sin\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \sin\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \sqrt{\frac{\pi}{2}}b^2 \cos$$

Antiderivative was successfully verified.

[In] Int[x^2*Sin[a + b*x - c*x^2],x]

[Out] (b*Cos[a + b*x - c*x^2])/(4*c^2) + (x*Cos[a + b*x - c*x^2])/(2*c) + (Sqrt[P
i/2]*Cos[a + b^2/(4*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3
/2)) + (b^2*Sqrt[Pi/2]*Cos[a + b^2/(4*c)]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqr
t[2*Pi])])/(4*c^(5/2)) - (b^2*Sqrt[Pi/2]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt
[2*Pi])]*Sin[a + b^2/(4*c)])/(4*c^(5/2)) + (Sqrt[Pi/2]*FresnelS[(b - 2*c*x)
/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^2/(4*c)])/(2*c^(3/2))

Rule 3463

Int[((d_.) + (e_.)*(x_))^(m_)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> -Simp[(e*(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2])/(2*c), x] + (Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Cos[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2 \sin\left(a + bx - cx^2\right) dx = \frac{x \cos\left(a + bx - cx^2\right)}{2c} - \frac{\int \cos\left(a + bx - cx^2\right) dx}{2c} + \frac{b \int x \sin\left(a + bx - cx^2\right) dx}{2c}$$

$$= \frac{b \cos\left(a + bx - cx^2\right)}{4c^2} + \frac{x \cos\left(a + bx - cx^2\right)}{2c} + \frac{b^2 \int \sin\left(a + bx - cx^2\right) dx}{4c^2} - \frac{\cos\left(a + \frac{b^2}{4c}\right) \int \cdots}{2 \cdots}$$

$$= \frac{b \cos\left(a + bx - cx^2\right)}{4c^2} + \frac{x \cos\left(a + bx - cx^2\right)}{2c} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a + \frac{b^2}{4c}\right) C\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^3 \cdots}$$

$$= \frac{b \cos\left(a + bx - cx^2\right)}{4c^2} + \frac{x \cos\left(a + bx - cx^2\right)}{2c} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a + \frac{b^2}{4c}\right) C\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{b^2 \sqrt{\frac{\pi}{2}} \cos\left(a - \cdots\right)}{4 \cdots}$$

Mathematica [A]   time = 0.711821, size = 165, normalized size = 0.66

$$\frac{-\sqrt{2\pi}\text{FresnelC}\left(\frac{2cx-b}{\sqrt{2\pi}\sqrt{c}}\right)\left(2c \cos\left(a + \frac{b^2}{4c}\right) - b^2 \sin\left(a + \frac{b^2}{4c}\right)\right) - \sqrt{2\pi}S\left(\frac{2cx-b}{\sqrt{c}\sqrt{2\pi}}\right)\left(2c \sin\left(a + \frac{b^2}{4c}\right) + b^2 \cos\left(a + \frac{b^2}{4c}\right)\right) + 2\sqrt{c}(b \cdots}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x - c*x^2],x]
```

```
[Out] (2*Sqrt[c]*(b + 2*c*x)*Cos[a + x*(b - c*x)] - Sqrt[2*Pi]*FresnelC[(-b + 2*c
*x)/(Sqrt[c]*Sqrt[2*Pi])]*(2*c*Cos[a + b^2/(4*c)] - b^2*Sin[a + b^2/(4*c)])
 - Sqrt[2*Pi]*FresnelS[(-b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*(b^2*Cos[a + b^2/
(4*c)] + 2*c*Sin[a + b^2/(4*c)]))/(8*c^(5/2))
```

Maple [A]   time = 0.01, size = 202, normalized size = 0.8

$$\frac{x \cos\left(-cx^2 + bx + a\right)}{2c} - \frac{b}{2c}\left(-\frac{\cos\left(-cx^2 + bx + a\right)}{2c} + \frac{b\sqrt{2}\sqrt{\pi}}{4}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} + ca\right)\right)\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx - \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(-c*x^2+b*x+a),x)
```

```
[Out] 1/2*x*cos(-c*x^2+b*x+a)/c-1/2*b/c*(-1/2*cos(-c*x^2+b*x+a)/c+1/4*b/c^(3/2)*2
^(1/2)*Pi^(1/2)*(cos((1/4*b^2+c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*
x-1/2*b))-sin((1/4*b^2+c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x-1/2*b
```

```
)))))-1/4/c^(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2+c*a)/c)*FresnelC(2^(1/2)/Pi
^(1/2)/c^(1/2)*(c*x-1/2*b))+sin((1/4*b^2+c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/
c^(1/2)*(c*x-1/2*b)))
```

Maxima [C]   time = 2.28549, size = 4122, normalized size = 16.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(-c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/16*((4*b*c*(e^(1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + e^(-1/4*(4*I*c^
2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/4*(b^2 + 4*a*c)/c) + b*c*(-4*I*
e^(1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + 4*I*e^(-1/4*(4*I*c^2*x^2 - 4*
I*b*c*x + I*b^2)/c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*((4*c^2*x^2 - 4*b*c*x
 + b^2)/abs(c))^(3/2) + (b^3*(-4*I*gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x
+ I*b^2)/c) + 4*I*gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs
(c)*cos(1/4*(b^2 + 4*a*c)/c) + 4*b^3*(gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c
*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(
c)*sin(1/4*(b^2 + 4*a*c)/c) + (c^3*(32*I*gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c) - 32*I*gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/
c))*abs(c)*cos(1/4*(b^2 + 4*a*c)/c) - 32*c^3*(gamma(3/2, 1/4*(4*I*c^2*x^2 -
 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/
c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*x^3 + (b*c^2*(-48*I*gamma(3/2, 1/4*(4*
I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + 48*I*gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I
*b*c*x + I*b^2)/c))*abs(c)*cos(1/4*(b^2 + 4*a*c)/c) + 48*b*c^2*(gamma(3/2,
1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 - 4
*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*x^2 + (b^2*c*(24*I*g
amma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) - 24*I*gamma(3/2, -1/4*(
4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/4*(b^2 + 4*a*c)/c) - 24*b
^2*c*(gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4
*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*x)*
cos(3/2*arctan2((4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)) + ((I*sqrt(pi)*(erf(1/2*
sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - I*sqrt(pi)*(erf(1/2*sqrt(
-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(1/4*(b^2 + 4*a*c)/c) -
 (sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(
pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(1/4*
(b^2 + 4*a*c)/c) + ((-8*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x +
 I*b^2)/c)) - 1) + 8*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I
*b^2)/c)) - 1))*b^2*c^3*cos(1/4*(b^2 + 4*a*c)/c) + 8*(sqrt(pi)*(erf(1/2*sqr
t((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I
```

```
*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*sin(1/4*(b^2 + 4*a*c)/c))*x
^3 + ((12*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) -
1) - 12*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1
))*b^3*c^2*cos(1/4*(b^2 + 4*a*c)/c) - 12*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x
^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4
*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(1/4*(b^2 + 4*a*c)/c))*x^2 + ((-6*I*
sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 6*I*sqr
t(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(
1/4*(b^2 + 4*a*c)/c) + 6*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x +
 I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2
)/c)) - 1))*b^4*c*sin(1/4*(b^2 + 4*a*c)/c))*x)*cos(1/2*arctan2((4*c^2*x^2 -
 4*b*c*x + b^2)/c, 0)) - (4*b^3*(gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x +
I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*co
s(1/4*(b^2 + 4*a*c)/c) - b^3*(-4*I*gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x
+ I*b^2)/c) + 4*I*gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs
(c)*sin(1/4*(b^2 + 4*a*c)/c) - (32*c^3*(gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b
*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*ab
s(c)*cos(1/4*(b^2 + 4*a*c)/c) + c^3*(32*I*gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I
*b*c*x + I*b^2)/c) - 32*I*gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)
/c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*x^3 + (48*b*c^2*(gamma(3/2, 1/4*(4*I*
c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 - 4*I*b*c*x
+ I*b^2)/c))*abs(c)*cos(1/4*(b^2 + 4*a*c)/c) - b*c^2*(-48*I*gamma(3/2, 1/4*
(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + 48*I*gamma(3/2, -1/4*(4*I*c^2*x^2 -
4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*x^2 - (24*b^2*c*(ga
mma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^
2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/4*(b^2 + 4*a*c)/c) + b^2*c*(24*
I*gamma(3/2, 1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) - 24*I*gamma(3/2, -1/
4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/4*(b^2 + 4*a*c)/c))*x)
*sin(3/2*arctan2((4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)) + ((sqrt(pi)*(erf(1/2*s
qrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4
*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(1/4*(b^2 + 4*a*c)/c) + (I
*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - I*sqrt
(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(1/4
*(b^2 + 4*a*c)/c) - (8*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I
*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/
c)) - 1))*b^2*c^3*cos(1/4*(b^2 + 4*a*c)/c) - (-8*I*sqrt(pi)*(erf(1/2*sqrt((
4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 8*I*sqrt(pi)*(erf(1/2*sqrt(-(4*
I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*sin(1/4*(b^2 + 4*a*c)/c))*
x^3 + (12*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1
) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^3
*c^2*cos(1/4*(b^2 + 4*a*c)/c) + (12*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 -
 4*I*b*c*x + I*b^2)/c)) - 1) - 12*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 -
4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(1/4*(b^2 + 4*a*c)/c))*x^2 - (6*(sq
rt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*
(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(1/4*(b
```

```
^2 + 4*a*c)/c) - (-6*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*
b^2)/c)) - 1) + 6*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^
2)/c)) - 1))*b^4*c*sin(1/4*(b^2 + 4*a*c)/c))*x)*sin(1/2*arctan2((4*c^2*x^2
- 4*b*c*x + b^2)/c, 0)))/(c^3*((4*c^2*x^2 - 4*b*c*x + b^2)/abs(c))^(3/2)*ab
s(c))
```

Fricas [A]   time = 1.60668, size = 443, normalized size = 1.76

$$\frac{\sqrt{2}\left(\pi b^2 \sin\left(\frac{b^2+4ac}{4c}\right) - 2\pi c \cos\left(\frac{b^2+4ac}{4c}\right)\right)\sqrt{\frac{c}{\pi}}\,C\left(\frac{\sqrt{2}(2cx-b)\sqrt{\frac{c}{\pi}}}{2c}\right) - \sqrt{2}\left(\pi b^2 \cos\left(\frac{b^2+4ac}{4c}\right) + 2\pi c \sin\left(\frac{b^2+4ac}{4c}\right)\right)\sqrt{\frac{c}{\pi}}\,S\left(\frac{\sqrt{2}(2cx}{2}\right)}{8c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(-c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(pi*b^2*sin(1/4*(b^2 + 4*a*c)/c) - 2*pi*c*cos(1/4*(b^2 + 4*a*c
)/c))*sqrt(c/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x - b)*sqrt(c/pi)/c) - sqrt(2
)*(pi*b^2*cos(1/4*(b^2 + 4*a*c)/c) + 2*pi*c*sin(1/4*(b^2 + 4*a*c)/c))*sqrt(
c/pi)*fresnel_sin(1/2*sqrt(2)*(2*c*x - b)*sqrt(c/pi)/c) + 2*(2*c^2*x + b*c)
*cos(c*x^2 - b*x - a))/c^3
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin\left(a + bx - cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(-c*x**2+b*x+a),x)
```

```
[Out] Integral(x**2*sin(a + b*x - c*x**2), x)
```

Giac [C]   time = 1.73842, size = 309, normalized size = 1.23

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}(b^2+2ic)\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2+4iac}{4c}\right)}}{\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}} - 2i\left(c\left(-2ix+\frac{ib}{c}\right)-2ib\right)e^{(icx^2-ibx-ia)}}{16c^2} - \frac{i\sqrt{2}\sqrt{\pi}(b^2-2ic)\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{b}{c}\right)\right.}{\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(-c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(I*sqrt(2)*sqrt(pi)*(b^2 + 2*I*c)*erf(-1/4*sqrt(2)*(2*x - b/c)*(-I*c/
abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(I*b^2 + 4*I*a*c)/c)/((-I*c/abs(c) + 1)*s
qrt(abs(c))) - 2*I*(c*(-2*I*x + I*b/c) - 2*I*b)*e^(I*c*x^2 - I*b*x - I*a))/
c^2 - 1/16*(-I*sqrt(2)*sqrt(pi)*(b^2 - 2*I*c)*erf(-1/4*sqrt(2)*(2*x - b/c)*
(I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 - 4*I*a*c)/c)/((I*c/abs(c) +
 1)*sqrt(abs(c))) - 2*I*(c*(-2*I*x + I*b/c) - 2*I*b)*e^(-I*c*x^2 + I*b*x +
I*a))/c^2
```

3.7 $\int x \sin\left(a + bx - cx^2\right) dx$

Optimal. Leaf size=124

$$-\frac{\sqrt{\frac{\pi}{2}} b \sin\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} b \cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{\cos\left(a + bx - cx^2\right)}{2c}$$

```
[Out] Cos[a + b*x - c*x^2]/(2*c) + (b*Sqrt[Pi/2]*Cos[a + b^2/(4*c)]*FresnelS[(b -
 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) - (b*Sqrt[Pi/2]*FresnelC[(b - 2*
c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^2/(4*c)])/(2*c^(3/2))
```

Rubi [A]   time = 0.0444542, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3461, 3447, 3351, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}} b \sin\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} b \cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{\cos\left(a + bx - cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x - c*x^2],x]
```

```
[Out] Cos[a + b*x - c*x^2]/(2*c) + (b*Sqrt[Pi/2]*Cos[a + b^2/(4*c)]*FresnelS[(b -
 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) - (b*Sqrt[Pi/2]*FresnelC[(b - 2*
c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^2/(4*c)])/(2*c^(3/2))
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x \sin\left(a + bx - cx^2\right) dx &= \frac{\cos\left(a + bx - cx^2\right)}{2c} + \frac{b \int \sin\left(a + bx - cx^2\right) dx}{2c} \\
&= \frac{\cos\left(a + bx - cx^2\right)}{2c} - \frac{\left(b\cos\left(a + \frac{b^2}{4c}\right)\right) \int \sin\left(\frac{(b-2cx)^2}{4c}\right) dx}{2c} + \frac{\left(b\sin\left(a + \frac{b^2}{4c}\right)\right) \int \cos\left(\frac{(b-2cx)^2}{4c}\right) dx}{2c} \\
&= \frac{\cos\left(a + bx - cx^2\right)}{2c} + \frac{b\sqrt{\frac{\pi}{2}}\cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{b\sqrt{\frac{\pi}{2}}C\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)\sin\left(a + \frac{b^2}{4c}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A]   time = 0.317877, size = 117, normalized size = 0.94

$$
\frac{\sqrt{2\pi}b\sin\left(a + \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{2cx-b}{\sqrt{2\pi}\sqrt{c}}\right) - \sqrt{2\pi}b\cos\left(a + \frac{b^2}{4c}\right)S\left(\frac{2cx-b}{\sqrt{c}\sqrt{2\pi}}\right) + 2\sqrt{c}\cos(a + x(b - cx))}{4c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x - c*x^2],x]
```

```
[Out] (2*Sqrt[c]*Cos[a + x*(b - c*x)] - b*Sqrt[2*Pi]*Cos[a + b^2/(4*c)]*FresnelS[
(-b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])] + b*Sqrt[2*Pi]*FresnelC[(-b + 2*c*x)/(Sq
rt[c]*Sqrt[2*Pi])]*Sin[a + b^2/(4*c)])/(4*c^(3/2))
```

Maple [A]   time = 0.008, size = 99, normalized size = 0.8

$$
\frac{\cos\left(-cx^2 + bx + a\right)}{2c} - \frac{b\sqrt{2}\sqrt{\pi}}{4}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} + ca\right)\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx - \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\left(\frac{b^2}{4} + ca\right)\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(-c*x^2+b*x+a),x)
```

```
[Out] 1/2*cos(-c*x^2+b*x+a)/c-1/4*b/c^(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2+c*a)/c
)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x-1/2*b))-sin((1/4*b^2+c*a)/c)*Fresn
elC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x-1/2*b)))
```

Maxima [C]   time = 1.70803, size = 1310, normalized size = 10.56

$$\text{result too large to display}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(((I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1)
- I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2
*cos(1/4*(b^2 + 4*a*c)/c) - (sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*
x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*
b^2)/c)) - 1))*b^2*sin(1/4*(b^2 + 4*a*c)/c) + ((-2*I*sqrt(pi)*(erf(1/2*sqrt
((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(erf(1/2*sqrt(-(
4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*cos(1/4*(b^2 + 4*a*c)/c) + 2
*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(
pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(1/4*
(b^2 + 4*a*c)/c))*x)*cos(1/2*arctan2((4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)) + (
(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(p
i)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(1/4*(
b^2 + 4*a*c)/c) + (I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^
2)/c)) - 1) - I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c
)) - 1))*b^2*sin(1/4*(b^2 + 4*a*c)/c) - (2*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2
*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 -
 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*cos(1/4*(b^2 + 4*a*c)/c) - (-2*I*sqrt(pi)
*(erf(1/2*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(e
rf(1/2*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(1/4*(b^2 +
 4*a*c)/c))*x)*sin(1/2*arctan2((4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)) + (2*c*(e
^(1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + e^(-1/4*(4*I*c^2*x^2 - 4*I*b*c
*x + I*b^2)/c))*cos(1/4*(b^2 + 4*a*c)/c) + c*(-2*I*e^(1/4*(4*I*c^2*x^2 - 4*
I*b*c*x + I*b^2)/c) + 2*I*e^(-1/4*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*sin
(1/4*(b^2 + 4*a*c)/c))*sqrt((4*c^2*x^2 - 4*b*c*x + b^2)/abs(c)))/(c^2*sqrt(
(4*c^2*x^2 - 4*b*c*x + b^2)/abs(c)))
```

Fricas [A]   time = 1.52921, size = 323, normalized size = 2.6

$$-\frac{\sqrt{2}\pi b\sqrt{\frac{c}{\pi}}\cos\left(\frac{b^2+4\,ac}{4\,c}\right)S\left(\frac{\sqrt{2}(2\,cx-b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\pi b\sqrt{\frac{c}{\pi}}\,C\left(\frac{\sqrt{2}(2\,cx-b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)\sin\left(\frac{b^2+4\,ac}{4\,c}\right)-2\,c\cos\left(cx^2-bx-a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*pi*b*sqrt(c/pi)*cos(1/4*(b^2 + 4*a*c)/c)*fresnel_sin(1/2*sqrt
(2)*(2*c*x - b)*sqrt(c/pi)/c) - sqrt(2)*pi*b*sqrt(c/pi)*fresnel_cos(1/2*sqr
t(2)*(2*c*x - b)*sqrt(c/pi)/c)*sin(1/4*(b^2 + 4*a*c)/c) - 2*c*cos(c*x^2 - b
*x - a))/c^2
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x\sin\left(a+bx-cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x**2+b*x+a),x)
```

```
[Out] Integral(x*sin(a + b*x - c*x**2), x)
```

Giac [C]   time = 2.44643, size = 250, normalized size = 2.02

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2\,x-\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2+4i\,ac}{4\,c}\right)}}{\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}-2\,e^{\left(icx^2-ibx-ia\right)}}{8\,c}-\frac{\frac{i\sqrt{2}\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{4}\sqrt{2}\left(2\,x-\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4i\,ac}{4\,c}\right)}}{\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}-2\,e^{\left(-icx^2+ibx+i\right.}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(I*sqrt(2)*sqrt(pi)*b*erf(-1/4*sqrt(2)*(2*x - b/c)*(-I*c/abs(c) + 1)*s
qrt(abs(c)))*e^(-1/4*(I*b^2 + 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

```
 - 2*e^(I*c*x^2 - I*b*x - I*a))/c - 1/8*(-I*sqrt(2)*sqrt(pi)*b*erf(-1/4*sqrt
(2)*(2*x - b/c)*(I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 - 4*I*a*c)/c
)/((I*c/abs(c) + 1)*sqrt(abs(c))) - 2*e^(-I*c*x^2 + I*b*x + I*a))/c
```

3.8 $\int \sin\left(a + bx - cx^2\right) dx$

Optimal. Leaf size=98

$$\frac{\sqrt{\frac{\pi}{2}}\cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}} - \frac{\sqrt{\frac{\pi}{2}}\sin\left(a + \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{\sqrt{c}}$$

[Out] (Sqrt[Pi/2]*Cos[a + b^2/(4*c)]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/
Sqrt[c] - (Sqrt[Pi/2]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^
2/(4*c)])/Sqrt[c]

Rubi [A]   time = 0.0248944, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3447, 3351, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}} - \frac{\sqrt{\frac{\pi}{2}}\sin\left(a + \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x - c*x^2],x]

[Out] (Sqrt[Pi/2]*Cos[a + b^2/(4*c)]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/
Sqrt[c] - (Sqrt[Pi/2]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^
2/(4*c)])/Sqrt[c]

Rule 3447

Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a + bx - cx^2\right) dx = -\left(\cos\left(a + \frac{b^2}{4c}\right) \int \sin\left(\frac{(b - 2cx)^2}{4c}\right) dx\right) + \sin\left(a + \frac{b^2}{4c}\right) \int \cos\left(\frac{(b - 2cx)^2}{4c}\right) dx$$

$$= \frac{\sqrt{\frac{\pi}{2}} \cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}} - \frac{\sqrt{\frac{\pi}{2}} C\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right) \sin\left(a + \frac{b^2}{4c}\right)}{\sqrt{c}}$$

Mathematica [A]   time = 0.185248, size = 89, normalized size = 0.91

$$\frac{\sqrt{\frac{\pi}{2}} \left(\sin\left(a + \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{2cx-b}{\sqrt{2\pi}\sqrt{c}}\right) - \cos\left(a + \frac{b^2}{4c}\right) S\left(\frac{2cx-b}{\sqrt{c}\sqrt{2\pi}}\right)\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x - c*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*(-(Cos[a + b^2/(4*c)]*FresnelS[(-b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi]
)]) + FresnelC[(-b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a + b^2/(4*c)]))/Sqrt
[c]
```

Maple [A]   time = 0.009, size = 80, normalized size = 0.8

$$-\frac{\sqrt{2}\sqrt{\pi}}{2}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} + ca\right)\right) \text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx - \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\left(\frac{b^2}{4} + ca\right)\right) \text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx - \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right)\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(-c*x^2+b*x+a),x)
```

```
[Out] -1/2*2^(1/2)*Pi^(1/2)/c^(1/2)*(cos((1/4*b^2+c*a)/c)*FresnelS(2^(1/2)/Pi^(1/
2)/c^(1/2)*(c*x-1/2*b))-sin((1/4*b^2+c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1
/2)*(c*x-1/2*b)))
```

Maxima [C]   time = 2.04943, size = 393, normalized size = 4.01

$$-\frac{\sqrt{\pi}\left(\left(\left(i\,\cos\left(\frac{1}{4}\,\pi+\frac{1}{2}\,\arctan(0,c)\right)+i\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,\arctan(0,c)\right)+\sin\left(\frac{1}{4}\,\pi+\frac{1}{2}\,\arctan(0,c)\right)-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,\text{arc}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(-c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((I*cos(1/4*pi + 1/2*arctan2(0, c)) + I*cos(-1/4*pi + 1/2*ar
ctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arctan2(
0, c)))*cos(1/4*(b^2 + 4*a*c)/c) - (cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-
1/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/
4*pi + 1/2*arctan2(0, c)))*sin(1/4*(b^2 + 4*a*c)/c))*erf(1/2*(2*I*c*x - I*b
)/sqrt(I*c)) + ((I*cos(1/4*pi + 1/2*arctan2(0, c)) + I*cos(-1/4*pi + 1/2*ar
ctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*pi + 1/2*arctan2(
0, c)))*cos(1/4*(b^2 + 4*a*c)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-
1/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/
4*pi + 1/2*arctan2(0, c)))*sin(1/4*(b^2 + 4*a*c)/c))*erf(1/2*(2*I*c*x - I*b
)/sqrt(-I*c)))/sqrt(abs(c))
```

Fricas [A]   time = 1.58511, size = 278, normalized size = 2.84

$$-\frac{\sqrt{2}\pi\sqrt{\frac{c}{\pi}}\cos\left(\frac{b^2+4\,ac}{4\,c}\right)S\left(\frac{\sqrt{2}(2\,cx-b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)-\sqrt{2}\pi\sqrt{\frac{c}{\pi}}\,C\left(\frac{\sqrt{2}(2\,cx-b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)\sin\left(\frac{b^2+4\,ac}{4\,c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(-c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*pi*sqrt(c/pi)*cos(1/4*(b^2 + 4*a*c)/c)*fresnel_sin(1/2*sqrt(2
)*(2*c*x - b)*sqrt(c/pi)/c) - sqrt(2)*pi*sqrt(c/pi)*fresnel_cos(1/2*sqrt(2)
*(2*c*x - b)*sqrt(c/pi)/c)*sin(1/4*(b^2 + 4*a*c)/c))/c
```

Sympy [A]   time = 0.545355, size = 94, normalized size = 0.96

$$\frac{\sqrt{2}\sqrt{\pi}\sqrt{-\frac{1}{c}}\left(\sin\left(a+\frac{b^2}{4c}\right)C\left(\frac{\sqrt{2}(b-2cx)}{2\sqrt{\pi}\sqrt{-c}}\right)+\cos\left(a+\frac{b^2}{4c}\right)S\left(\frac{\sqrt{2}(b-2cx)}{2\sqrt{\pi}\sqrt{-c}}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x**2+b*x+a),x)
```

```
[Out] sqrt(2)*sqrt(pi)*sqrt(-1/c)*(sin(a + b**2/(4*c))*fresnelc(sqrt(2)*(b - 2*c*
x)/(2*sqrt(pi)*sqrt(-c))) + cos(a + b**2/(4*c))*fresnels(sqrt(2)*(b - 2*c*x
)/(2*sqrt(pi)*sqrt(-c))))/2
```

Giac [C]   time = 2.79534, size = 185, normalized size = 1.89

$$-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2+4iac}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}+\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x-\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2-4iac}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x - b/c)*(-I*c/abs(c) + 1)*sqrt
(abs(c)))*e^(-1/4*(I*b^2 + 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) + 1
/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x - b/c)*(I*c/abs(c) + 1)*sqrt(ab
s(c)))*e^(-1/4*(-I*b^2 - 4*I*a*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.9 $\int \dfrac{\sin\left(a+bx-cx^2\right)}{x}\,dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\sin\left(a+bx-cx^2\right)}{x}, x\right)$$

[Out] Unintegrable[Sin[a + b*x - c*x^2]/x, x]

Rubi [A]   time = 0.0081506, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin\left(a+bx-cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[a + b*x - c*x^2]/x,x]

[Out] Defer[Int][Sin[a + b*x - c*x^2]/x, x]

Rubi steps

$$\int \frac{\sin\left(a+bx-cx^2\right)}{x}\,dx = \int \frac{\sin\left(a+bx-cx^2\right)}{x}\,dx$$

Mathematica [A]   time = 4.7916, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bx-cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + b*x - c*x^2]/x,x]

[Out] Integrate[Sin[a + b*x - c*x^2]/x, x]

Maple [A]   time = 0.046, size = 0, normalized size = 0.

$$\int \frac{\sin\left(-cx^2 + bx + a\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(-c*x^2+b*x+a)/x,x)
```

```
[Out] int(sin(-c*x^2+b*x+a)/x,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin\left(cx^2 - bx - a\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)/x,x, algorithm="maxima")
```

```
[Out] -integrate(sin(c*x^2 - b*x - a)/x, x)
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sin\left(cx^2 - bx - a\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)/x,x, algorithm="fricas")
```

```
[Out] integral(-sin(c*x^2 - b*x - a)/x, x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + bx - cx^2\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x**2+b*x+a)/x,x)
```

```
[Out] Integral(sin(a + b*x - c*x**2)/x, x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(-cx^2 + bx + a\right)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)/x,x, algorithm="giac")
```

```
[Out] integrate(sin(-c*x^2 + b*x + a)/x, x)
```

3.10 $\int \left(-\dfrac{b\cos\left(a+bx-cx^2\right)}{x} + \dfrac{\sin\left(a+bx-cx^2\right)}{x^2} \right) dx$

Optimal. Leaf size=110

$$\sqrt{2\pi}\sqrt{c}\cos\left(a+\frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right) + \sqrt{2\pi}\sqrt{c}\sin\left(a+\frac{b^2}{4c}\right)S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right) - \frac{\sin\left(a+bx-cx^2\right)}{x}$$

```
[Out] Sqrt[c]*Sqrt[2*Pi]*Cos[a + b^2/(4*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*
Pi])] + Sqrt[c]*Sqrt[2*Pi]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a
 + b^2/(4*c)] - Sin[a + b*x - c*x^2]/x
```

Rubi [A]   time = 0.084384, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3465, 3448, 3352, 3351}

$$\sqrt{2\pi}\sqrt{c}\cos\left(a+\frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{2\pi}\sqrt{c}}\right) + \sqrt{2\pi}\sqrt{c}\sin\left(a+\frac{b^2}{4c}\right)S\left(\frac{b-2cx}{\sqrt{c}\sqrt{2\pi}}\right) - \frac{\sin\left(a+bx-cx^2\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-((b*Cos[a + b*x - c*x^2])/x) + Sin[a + b*x - c*x^2]/x^2,x]
```

```
[Out] Sqrt[c]*Sqrt[2*Pi]*Cos[a + b^2/(4*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*
Pi])] + Sqrt[c]*Sqrt[2*Pi]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a
 + b^2/(4*c)] - Sin[a + b*x - c*x^2]/x
```

Rule 3465

```
Int[((d_.) + (e_.)*(x_))^(m_)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> Simp[((d + e*x)^(m + 1)*Sin[a + b*x + c*x^2])/(e*(m + 1)), x] + (-D
ist[(2*c)/(e^2*(m + 1)), Int[(d + e*x)^(m + 2)*Cos[a + b*x + c*x^2], x], x]
 - Dist[(b*e - 2*c*d)/(e^2*(m + 1)), Int[(d + e*x)^(m + 1)*Cos[a + b*x + c*
x^2], x], x]) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && LtQ[m,
 -1]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
```

```
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \left(-\frac{b\cos\left(a + bx - cx^2\right)}{x} + \frac{\sin\left(a + bx - cx^2\right)}{x^2} \right) dx = -\left(b \int \frac{\cos\left(a + bx - cx^2\right)}{x} dx \right) + \int \frac{\sin\left(a + bx - cx^2\right)}{x^2} dx$$

$$= -\frac{\sin\left(a + bx - cx^2\right)}{x} - (2c)\int \cos\left(a + bx - cx^2\right) dx$$

$$= -\frac{\sin\left(a + bx - cx^2\right)}{x} - \left(2c\cos\left(a + \frac{b^2}{4c}\right)\right)\int \cos\left(\frac{(b - 2cx)^2}{4c}\right) dx - $$

$$= \sqrt{c}\sqrt{2\pi}\cos\left(a + \frac{b^2}{4c}\right) C\left(\frac{b - 2cx}{\sqrt{c}\sqrt{2\pi}}\right) + \sqrt{c}\sqrt{2\pi}S\left(\frac{b - 2cx}{\sqrt{c}\sqrt{2\pi}}\right)\sin\left(a + \frac{b}{4}\right)$$

Mathematica [A]   time = 0.802383, size = 115, normalized size = 1.05

$$-\frac{\sqrt{2\pi}\sqrt{c}x\cos\left(a + \frac{b^2}{4c}\right)\text{FresnelC}\left(\frac{2cx-b}{\sqrt{2\pi}\sqrt{c}}\right) + \sqrt{2\pi}\sqrt{c}x\sin\left(a + \frac{b^2}{4c}\right)S\left(\frac{2cx-b}{\sqrt{c}\sqrt{2\pi}}\right) + \sin(a + x(b - cx))}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((b*Cos[a + b*x - c*x^2])/x) + Sin[a + b*x - c*x^2]/x^2,x]
```

```
[Out] -((Sqrt[c]*Sqrt[2*Pi]*x*Cos[a + b^2/(4*c)]*FresnelC[(-b + 2*c*x)/(Sqrt[c]*S
qrt[2*Pi])] + Sqrt[c]*Sqrt[2*Pi]*x*FresnelS[(-b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi
])]*Sin[a + b^2/(4*c)] + Sin[a + x*(b - c*x)])/x)
```

Maple [F]   time = 0.204, size = 0, normalized size = 0.

$$\int -\frac{b\cos\left(-cx^2 + bx + a\right)}{x} + \frac{\sin\left(-cx^2 + bx + a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-b*cos(-c*x^2+b*x+a)/x+sin(-c*x^2+b*x+a)/x^2,x)

[Out] int(-b*cos(-c*x^2+b*x+a)/x+sin(-c*x^2+b*x+a)/x^2,x)

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\cos\left(cx^2 - bx - a\right)}{x} - \frac{\sin\left(cx^2 - bx - a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-b*cos(-c*x^2+b*x+a)/x+sin(-c*x^2+b*x+a)/x^2,x, algorithm="maxima
")

[Out] integrate(-b*cos(c*x^2 - b*x - a)/x - sin(c*x^2 - b*x - a)/x^2, x)

Fricas [A]   time = 1.58258, size = 309, normalized size = 2.81

$$-\frac{\sqrt{2}\pi x\sqrt{\frac{c}{\pi}}\cos\left(\frac{b^2+4\,ac}{4c}\right)C\left(\frac{\sqrt{2}(2\,cx-b)\sqrt{\frac{c}{\pi}}}{2c}\right) + \sqrt{2}\pi x\sqrt{\frac{c}{\pi}}S\left(\frac{\sqrt{2}(2\,cx-b)\sqrt{\frac{c}{\pi}}}{2c}\right)\sin\left(\frac{b^2+4\,ac}{4c}\right) - \sin\left(cx^2 - bx - a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-b*cos(-c*x^2+b*x+a)/x+sin(-c*x^2+b*x+a)/x^2,x, algorithm="fricas
")

[Out] -(sqrt(2)*pi*x*sqrt(c/pi)*cos(1/4*(b^2 + 4*a*c)/c)*fresnel_cos(1/2*sqrt(2)*
(2*c*x - b)*sqrt(c/pi)/c) + sqrt(2)*pi*x*sqrt(c/pi)*fresnel_sin(1/2*sqrt(2)
*(2*c*x - b)*sqrt(c/pi)/c)*sin(1/4*(b^2 + 4*a*c)/c) - sin(c*x^2 - b*x - a))
/x

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sin\left(a+bx-cx^2\right)}{x^2}\,dx - \int \frac{b\cos\left(a+bx-cx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*cos(-c*x**2+b*x+a)/x+sin(-c*x**2+b*x+a)/x**2,x)
```

```
[Out] -Integral(-sin(a + b*x - c*x**2)/x**2, x) - Integral(b*cos(a + b*x - c*x**2
)/x, x)
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int -\frac{b\cos\left(-cx^2+bx+a\right)}{x} + \frac{\sin\left(-cx^2+bx+a\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-b*cos(-c*x^2+b*x+a)/x+sin(-c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(-b*cos(-c*x^2 + b*x + a)/x + sin(-c*x^2 + b*x + a)/x^2, x)
```

3.11 $\quad \int x^2 \sin\left(\frac{1}{4} + x + x^2\right) dx$

Optimal. Leaf size=82

$$\frac{1}{2}\sqrt{\frac{\pi}{2}}\mathrm{FresnelC}\left(\frac{2x+1}{\sqrt{2\pi}}\right) + \frac{1}{4}\sqrt{\frac{\pi}{2}}S\left(\frac{2x+1}{\sqrt{2\pi}}\right) - \frac{1}{2}x\cos\left(x^2+x+\frac{1}{4}\right) + \frac{1}{4}\cos\left(x^2+x+\frac{1}{4}\right)$$

```
[Out] Cos[1/4 + x + x^2]/4 - (x*Cos[1/4 + x + x^2])/2 + (Sqrt[Pi/2]*FresnelC[(1 +
 2*x)/Sqrt[2*Pi]])/2 + (Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]])/4
```

Rubi [A] time = 0.0330391, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3463, 3446, 3352, 3461, 3445, 3351}

$$\frac{1}{2}\sqrt{\frac{\pi}{2}}\mathrm{FresnelC}\left(\frac{2x+1}{\sqrt{2\pi}}\right) + \frac{1}{4}\sqrt{\frac{\pi}{2}}S\left(\frac{2x+1}{\sqrt{2\pi}}\right) - \frac{1}{2}x\cos\left(x^2+x+\frac{1}{4}\right) + \frac{1}{4}\cos\left(x^2+x+\frac{1}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[1/4 + x + x^2],x]
```

```
[Out] Cos[1/4 + x + x^2]/4 - (x*Cos[1/4 + x + x^2])/2 + (Sqrt[Pi/2]*FresnelC[(1 +
 2*x)/Sqrt[2*Pi]])/2 + (Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]])/4
```

Rule 3463

```
Int[((d_.) + (e_.)*(x_))^(m_)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> -Simp[(e*(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2])/(2*c), x] + (Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Cos[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3446

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Cos[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3445

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Sin[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x^2 \sin\left(\frac{1}{4} + x + x^2\right) dx = -\frac{1}{2}x\cos\left(\frac{1}{4} + x + x^2\right) + \frac{1}{2}\int \cos\left(\frac{1}{4} + x + x^2\right) dx - \frac{1}{2}\int x\sin\left(\frac{1}{4} + x + x^2\right) dx$$

$$= \frac{1}{4}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{1}{2}x\cos\left(\frac{1}{4} + x + x^2\right) + \frac{1}{4}\int \sin\left(\frac{1}{4} + x + x^2\right) dx + \frac{1}{2}\int \cos\left(\frac{1}{4}(1 + 2x)\right)$$

$$= \frac{1}{4}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{1}{2}x\cos\left(\frac{1}{4} + x + x^2\right) + \frac{1}{2}\sqrt{\frac{\pi}{2}}C\left(\frac{1 + 2x}{\sqrt{2\pi}}\right) + \frac{1}{4}\int \sin\left(\frac{1}{4}(1 + 2x)^2\right) dx$$

$$= \frac{1}{4}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{1}{2}x\cos\left(\frac{1}{4} + x + x^2\right) + \frac{1}{2}\sqrt{\frac{\pi}{2}}C\left(\frac{1 + 2x}{\sqrt{2\pi}}\right) + \frac{1}{4}\sqrt{\frac{\pi}{2}}S\left(\frac{1 + 2x}{\sqrt{2\pi}}\right)$$

Mathematica [A]   time = 0.117096, size = 66, normalized size = 0.8

$$\frac{1}{8}\left(2\sqrt{2\pi}\text{FresnelC}\left(\frac{2x + 1}{\sqrt{2\pi}}\right) + \sqrt{2\pi}S\left(\frac{2x + 1}{\sqrt{2\pi}}\right) + 2(1 - 2x)\cos\left(x^2 + x + \frac{1}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[1/4 + x + x^2],x]
```

```
[Out] (2*(1 - 2*x)*Cos[1/4 + x + x^2] + 2*Sqrt[2*Pi]*FresnelC[(1 + 2*x)/Sqrt[2*Pi
]] + Sqrt[2*Pi]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]])/8
```

Maple [A]   time = 0.009, size = 59, normalized size = 0.7

$$-\frac{x}{2}\cos\left(\frac{1}{4}+x+x^2\right)+\frac{1}{4}\cos\left(\frac{1}{4}+x+x^2\right)+\frac{\sqrt{2}\sqrt{\pi}}{8}\mathrm{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(x+\frac{1}{2}\right)\right)+\frac{\sqrt{2}\sqrt{\pi}}{4}\mathrm{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(1/4+x+x^2),x)
```

```
[Out] -1/2*x*cos(1/4+x+x^2)+1/4*cos(1/4+x+x^2)+1/8*2^(1/2)*Pi^(1/2)*FresnelS(2^(1
/2)/Pi^(1/2)*(x+1/2))+1/4*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*(x+1/2
))
```

Maxima [C]   time = 3.97883, size = 211, normalized size = 2.57

$$512\,x\left(e^{\left(i\,x^2+i\,x+\frac{1}{4}i\right)}+e^{\left(-i\,x^2-i\,x-\frac{1}{4}i\right)}\right)+\sqrt{4\,x^2+4\,x+1}\left((32i+32)\,\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{i\,x^2+i\,x+\frac{1}{4}i}\right)-1\right)-(32i-32)\,\sqrt{2}\sqrt{\pi}\left(\epsilon\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(1/4+x+x^2),x, algorithm="maxima")
```

```
[Out] 1/1024*(512*x*(e^(I*x^2 + I*x + 1/4*I) + e^(-I*x^2 - I*x - 1/4*I)) + sqrt(4
*x^2 + 4*x + 1)*((32*I + 32)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*x^2 + I*x + 1/4*I
)) - 1) - (32*I - 32)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*x^2 - I*x - 1/4*I)) - 1
) + (128*I - 128)*sqrt(2)*gamma(3/2, I*x^2 + I*x + 1/4*I) - (128*I + 128)*s
qrt(2)*gamma(3/2, -I*x^2 - I*x - 1/4*I)) + 256*e^(I*x^2 + I*x + 1/4*I) + 25
6*e^(-I*x^2 - I*x - 1/4*I))/(2*x + 1)
```

Fricas [A]   time = 1.48286, size = 228, normalized size = 2.78

$$-\frac{1}{4}(2\,x-1)\cos\left(x^2+x+\frac{1}{4}\right)+\frac{1}{4}\,\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,x+1)}{2\,\sqrt{\pi}}\right)+\frac{1}{8}\,\sqrt{2}\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}(2\,x+1)}{2\,\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(1/4+x+x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x - 1)*cos(x^2 + x + 1/4) + 1/4*sqrt(2)*sqrt(pi)*fresnel_cos(1/2*sq
rt(2)*(2*x + 1)/sqrt(pi)) + 1/8*sqrt(2)*sqrt(pi)*fresnel_sin(1/2*sqrt(2)*(2
*x + 1)/sqrt(pi))
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin\left(x^2 + x + \frac{1}{4}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(1/4+x+x**2),x)
```

```
[Out] Integral(x**2*sin(x**2 + x + 1/4), x)
```

Giac [C]   time = 2.41814, size = 101, normalized size = 1.23

$$-\left(\frac{1}{32}i + \frac{3}{32}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{4}i - \frac{1}{4}\right)\sqrt{2}(2x+1)\right) + \left(\frac{1}{32}i - \frac{3}{32}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{4}i + \frac{1}{4}\right)\sqrt{2}(2x+1)\right) - \frac{1}{8}i\left(-2ix + i\right)e^{\left(ix^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(1/4+x+x^2),x, algorithm="giac")
```

```
[Out] -(1/32*I + 3/32)*sqrt(2)*sqrt(pi)*erf((1/4*I - 1/4)*sqrt(2)*(2*x + 1)) + (1
/32*I - 3/32)*sqrt(2)*sqrt(pi)*erf(-(1/4*I + 1/4)*sqrt(2)*(2*x + 1)) - 1/8*
I*(-2*I*x + I)*e^(I*x^2 + I*x + 1/4*I) - 1/8*I*(-2*I*x + I)*e^(-I*x^2 - I*x
 - 1/4*I)
```

3.12 $\int x \sin\left(\frac{1}{4} + x + x^2\right) dx$

Optimal. Leaf size=41

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}S\left(\frac{2x+1}{\sqrt{2\pi}}\right) - \frac{1}{2}\cos\left(x^2 + x + \frac{1}{4}\right)$$

[Out] -Cos[1/4 + x + x^2]/2 - (Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]])/2

Rubi [A]   time = 0.0129375, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3461, 3445, 3351}

$$-\frac{1}{2}\sqrt{\frac{\pi}{2}}S\left(\frac{2x+1}{\sqrt{2\pi}}\right) - \frac{1}{2}\cos\left(x^2 + x + \frac{1}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[1/4 + x + x^2],x]

[Out] -Cos[1/4 + x + x^2]/2 - (Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]])/2

Rule 3461

Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]

Rule 3445

Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Sin[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rule 3351

Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\int x \sin\left(\frac{1}{4} + x + x^2\right) dx = -\frac{1}{2}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{1}{2}\int \sin\left(\frac{1}{4} + x + x^2\right) dx$$

$$= -\frac{1}{2}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{1}{2}\int \sin\left(\frac{1}{4}(1 + 2x)^2\right) dx$$

$$= -\frac{1}{2}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{1}{2}\sqrt{\frac{\pi}{2}} S\left(\frac{1 + 2x}{\sqrt{2\pi}}\right)$$

Mathematica [A]   time = 0.0500704, size = 39, normalized size = 0.95

$$\frac{1}{4}\left(-\sqrt{2\pi}S\left(\frac{2x + 1}{\sqrt{2\pi}}\right) - 2\cos\left(x^2 + x + \frac{1}{4}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sin[1/4 + x + x^2],x]

[Out] (-2*Cos[1/4 + x + x^2] - Sqrt[2*Pi]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]])/4

Maple [A]   time = 0.009, size = 30, normalized size = 0.7

$$-\frac{1}{2}\cos\left(\frac{1}{4} + x + x^2\right) - \frac{\sqrt{2}\sqrt{\pi}}{4}\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(x + \frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*sin(1/4+x+x^2),x)

[Out] -1/2*cos(1/4+x+x^2)-1/4*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(x+1/2))

Maxima [C]   time = 4.14306, size = 166, normalized size = 4.05

$$-\frac{2048\,x\left(e^{\left(ix^2+ix+\frac{1}{4}i\right)} + e^{\left(-ix^2-ix-\frac{1}{4}i\right)}\right) - \sqrt{4x^2 + 4x + 1}\left(-(256i + 256)\sqrt{2}\sqrt{\pi}\left(\text{erf}\left(\sqrt{ix^2 + ix + \frac{1}{4}i}\right) - 1\right) + (256i - 256)\right.}{4096\,(2x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x^2),x, algorithm="maxima")
```

```
[Out] -1/4096*(2048*x*(e^(I*x^2 + I*x + 1/4*I) + e^(-I*x^2 - I*x - 1/4*I)) - sqrt
(4*x^2 + 4*x + 1)*(-(256*I + 256)*sqrt(2)*sqrt(pi)*(erf(sqrt(I*x^2 + I*x +
1/4*I)) - 1) + (256*I - 256)*sqrt(2)*sqrt(pi)*(erf(sqrt(-I*x^2 - I*x - 1/4*
I)) - 1)) + 1024*e^(I*x^2 + I*x + 1/4*I) + 1024*e^(-I*x^2 - I*x - 1/4*I))/(
2*x + 1)
```

Fricas [A]   time = 1.39032, size = 124, normalized size = 3.02

$$-\frac{1}{4}\,\sqrt{2}\sqrt{\pi}\,\mathrm{S}\!\left(\frac{\sqrt{2}(2\,x+1)}{2\,\sqrt{\pi}}\right)-\frac{1}{2}\,\cos\!\left(x^2+x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x^2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt(pi)*fresnel_sin(1/2*sqrt(2)*(2*x + 1)/sqrt(pi)) - 1/2*cos
(x^2 + x + 1/4)
```

Sympy [B]   time = 1.14774, size = 160, normalized size = 3.9

$$-\frac{3\sqrt{2}\sqrt{\pi}xS\!\left(\frac{\sqrt{2}x}{\sqrt{\pi}}+\frac{\sqrt{2}}{2\sqrt{\pi}}\right)\Gamma\!\left(\frac{3}{4}\right)}{8\Gamma\!\left(\frac{7}{4}\right)}+\frac{\sqrt{2}\sqrt{\pi}xS\!\left(\frac{\sqrt{2}x}{\sqrt{\pi}}+\frac{\sqrt{2}}{2\sqrt{\pi}}\right)}{2}-\frac{3\cos\!\left(\left(x+\frac{1}{2}\right)^2\right)\Gamma\!\left(\frac{3}{4}\right)}{8\Gamma\!\left(\frac{7}{4}\right)}-\frac{3\sqrt{2}\sqrt{\pi}S\!\left(\frac{\sqrt{2}x}{\sqrt{\pi}}+\frac{\sqrt{2}}{2\sqrt{\pi}}\right)\Gamma\!\left(\frac{3}{4}\right)}{16\Gamma\!\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x**2),x)
```

```
[Out] -3*sqrt(2)*sqrt(pi)*x*fresnels(sqrt(2)*x/sqrt(pi) + sqrt(2)/(2*sqrt(pi)))*g
amma(3/4)/(8*gamma(7/4)) + sqrt(2)*sqrt(pi)*x*fresnels(sqrt(2)*x/sqrt(pi) +
 sqrt(2)/(2*sqrt(pi)))/2 - 3*cos((x + 1/2)**2)*gamma(3/4)/(8*gamma(7/4)) -
3*sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*x/sqrt(pi) + sqrt(2)/(2*sqrt(pi)))*gamm
a(3/4)/(16*gamma(7/4))
```

Giac [C]   time = 1.29688, size = 88, normalized size = 2.15

$$-\left(\frac{1}{16}i - \frac{1}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{4}i - \frac{1}{4}\right)\sqrt{2}(2x+1)\right) + \left(\frac{1}{16}i + \frac{1}{16}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{4}i + \frac{1}{4}\right)\sqrt{2}(2x+1)\right) - \frac{1}{4}e^{\left(ix^2 + ix + \frac{1}{4}i\right)} - \frac{1}{4}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x^2),x, algorithm="giac")
```

```
[Out] -(1/16*I - 1/16)*sqrt(2)*sqrt(pi)*erf((1/4*I - 1/4)*sqrt(2)*(2*x + 1)) + (1
/16*I + 1/16)*sqrt(2)*sqrt(pi)*erf(-(1/4*I + 1/4)*sqrt(2)*(2*x + 1)) - 1/4*
e^(I*x^2 + I*x + 1/4*I) - 1/4*e^(-I*x^2 - I*x - 1/4*I)
```

3.13 $\quad \int \sin\left(\frac{1}{4} + x + x^2\right) dx$

Optimal. Leaf size=24

$$\sqrt{\frac{\pi}{2}} S\left(\frac{2x+1}{\sqrt{2\pi}}\right)$$

`[Out] Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]]`

Rubi [A]   time = 0.0060308, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3445, 3351}

$$\sqrt{\frac{\pi}{2}} S\left(\frac{2x+1}{\sqrt{2\pi}}\right)$$

Antiderivative was successfully verified.

`[In] Int[Sin[1/4 + x + x^2],x]`

`[Out] Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]]`

Rule 3445

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Sin[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(\frac{1}{4} + x + x^2\right) dx = \int \sin\left(\frac{1}{4}(1 + 2x)^2\right) dx$$

$$= \sqrt{\frac{\pi}{2}} S\left(\frac{1 + 2x}{\sqrt{2\pi}}\right)$$

Mathematica [A]   time = 0.0265427, size = 24, normalized size = 1.

$$\sqrt{\frac{\pi}{2}} S\left(\frac{2x+1}{\sqrt{2\pi}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[1/4 + x + x^2],x]

[Out] Sqrt[Pi/2]*FresnelS[(1 + 2*x)/Sqrt[2*Pi]]

Maple [A]   time = 0.007, size = 20, normalized size = 0.8

$$\frac{\sqrt{2}\sqrt{\pi}}{2} \mathrm{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(1/4+x+x^2),x)

[Out] 1/2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*(x+1/2))

Maxima [C]   time = 3.76987, size = 95, normalized size = 3.96

$$\frac{1}{16}\sqrt{\pi}\left((i+1)\sqrt{2}\operatorname{erf}\left(-\frac{1}{2}(-1)^{\frac{3}{4}}(2ix+i)\right)+(i+1)\sqrt{2}\operatorname{erf}\left(-\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}(2ix+i)\right)-(i-1)\sqrt{2}\operatorname{erf}\left(-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}(2i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/4+x+x^2),x, algorithm="maxima")

[Out] 1/16*sqrt(pi)*((I + 1)*sqrt(2)*erf(-1/2*(-1)^(3/4)*(2*I*x + I)) + (I + 1)*s
qrt(2)*erf(-(1/4*I - 1/4)*sqrt(2)*(2*I*x + I)) - (I - 1)*sqrt(2)*erf(-(1/4*
I + 1/4)*sqrt(2)*(2*I*x + I)) + (I - 1)*sqrt(2)*erf(1/2*(2*I*x + I)/sqrt(-I
)))

Fricas [A]   time = 1.35415, size = 89, normalized size = 3.71

$$\frac{1}{2} \sqrt{2}\sqrt{\pi}\, \mathrm{S}\left(\frac{\sqrt{2}(2\,x+1)}{2\,\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(pi)*fresnel_sin(1/2*sqrt(2)*(2*x + 1)/sqrt(pi))
```

Sympy [A]   time = 0.499589, size = 29, normalized size = 1.21

$$\frac{\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}(2x+1)}{2\sqrt{\pi}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x**2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*(2*x + 1)/(2*sqrt(pi)))/2
```

Giac [C]   time = 1.17521, size = 53, normalized size = 2.21

$$\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}(2\,x+1)\right)-\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}(2\,x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2),x, algorithm="giac")
```

```
[Out] (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/4*I - 1/4)*sqrt(2)*(2*x + 1)) - (1/8*
I + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/4*I + 1/4)*sqrt(2)*(2*x + 1))
```

3.14 $\int \dfrac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$

Optimal. Leaf size=15

$$\text{Unintegrable}\left(\frac{\sin\left(x^2+x+\frac{1}{4}\right)}{x},x\right)$$

[Out] Unintegrable[Sin[1/4 + x + x^2]/x, x]

Rubi [A]   time = 0.0080294, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[1/4 + x + x^2]/x,x]

[Out] Defer[Int][Sin[1/4 + x + x^2]/x, x]

Rubi steps

$$\int \frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}\,dx = \int \frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

Mathematica [A]   time = 14.4811, size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[1/4 + x + x^2]/x,x]

```
[Out] Integrate[Sin[1/4 + x + x^2]/x, x]
```

Maple [A]   time = 0.063, size = 0, normalized size = 0.

$$\int \frac{1}{x} \sin\left(\frac{1}{4} + x + x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4+x+x^2)/x,x)
```

```
[Out] int(sin(1/4+x+x^2)/x,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sin(x^2 + x + 1/4)/x, x)
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)/x,x, algorithm="fricas")
```

```
[Out] integral(sin(x^2 + x + 1/4)/x, x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/4+x+x**2)/x,x)

[Out] Integral(sin(x**2 + x + 1/4)/x, x)

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/4+x+x^2)/x,x, algorithm="giac")

[Out] integrate(sin(x^2 + x + 1/4)/x, x)

3.15 $\quad \int \dfrac{\sin\left(\frac{1}{4}+x+x^2\right)}{x^2}\,dx$

Optimal. Leaf size=52

$$\text{Unintegrable}\left(\frac{\cos\left(x^2+x+\frac{1}{4}\right)}{x},x\right)+\sqrt{2\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{2\pi}}\right)-\frac{\sin\left(x^2+x+\frac{1}{4}\right)}{x}$$

```
[Out] Sqrt[2*Pi]*FresnelC[(1 + 2*x)/Sqrt[2*Pi]] - Sin[1/4 + x + x^2]/x + Unintegr
able[Cos[1/4 + x + x^2]/x, x]
```

Rubi [A] $\quad$ time = 0.0246402, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$,
Rules used = {}

$$\int \frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[1/4 + x + x^2]/x^2,x]
```

```
[Out] Sqrt[2*Pi]*FresnelC[(1 + 2*x)/Sqrt[2*Pi]] - Sin[1/4 + x + x^2]/x + Defer[In
t][Cos[1/4 + x + x^2]/x, x]
```

Rubi steps

$$\int \frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x^2}\,dx = -\frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}+2\int \cos\left(\frac{1}{4}+x+x^2\right)dx+\int \frac{\cos\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

$$= -\frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}+2\int \cos\left(\frac{1}{4}(1+2x)^2\right)dx+\int \frac{\cos\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

$$= \sqrt{2\pi}C\left(\frac{1+2x}{\sqrt{2\pi}}\right)-\frac{\sin\left(\frac{1}{4}+x+x^2\right)}{x}+\int \frac{\cos\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

Mathematica [A]   time = 11.5538, size = 0, normalized size = 0.

$$\int \frac{\sin\left(\frac{1}{4} + x + x^2\right)}{x^2}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[1/4 + x + x^2]/x^2,x]
```

```
[Out] Integrate[Sin[1/4 + x + x^2]/x^2, x]
```

Maple [A]   time = 0.066, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sin\left(\frac{1}{4} + x + x^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4+x+x^2)/x^2,x)
```

```
[Out] int(sin(1/4+x+x^2)/x^2,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(x^2 + x + 1/4)/x^2, x)
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sin(x^2 + x + 1/4)/x^2, x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x**2)/x**2,x)
```

```
[Out] Integral(sin(x**2 + x + 1/4)/x**2, x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(x^2 + x + 1/4)/x^2, x)
```

3.16 $\quad \int x^2 \sin^2\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=248

$$\frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{3/2}} - \frac{\sqrt{\pi}b^2\cos\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}b^2\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} + \sqrt{\pi}\cos\left(2\right.$$

[Out] x^3/6 - (b^2*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sq
rt[Pi])])/(16*c^(5/2)) + (Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelS[(b + 2*c*x
)/(Sqrt[c]*Sqrt[Pi])])/(16*c^(3/2)) + (Sqrt[Pi]*FresnelC[(b + 2*c*x)/(Sqrt[
c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(16*c^(3/2)) + (b^2*Sqrt[Pi]*FresnelS[(
b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(16*c^(5/2)) + (b*Sin[
2*a + 2*b*x + 2*c*x^2])/(16*c^2) - (x*Sin[2*a + 2*b*x + 2*c*x^2])/(8*c)

Rubi [A] $\quad$ time = 0.227181, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3467, 3464, 3447, 3351, 3352, 3462, 3448}

$$\frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{3/2}} - \frac{\sqrt{\pi}b^2\cos\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}b^2\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} + \sqrt{\pi}\cos\left(2\right.$$

Antiderivative was successfully verified.

[In] Int[x^2*Sin[a + b*x + c*x^2]^2,x]

[Out] x^3/6 - (b^2*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sq
rt[Pi])])/(16*c^(5/2)) + (Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelS[(b + 2*c*x
)/(Sqrt[c]*Sqrt[Pi])])/(16*c^(3/2)) + (Sqrt[Pi]*FresnelC[(b + 2*c*x)/(Sqrt[
c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(16*c^(3/2)) + (b^2*Sqrt[Pi]*FresnelS[(
b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(16*c^(5/2)) + (b*Sin[
2*a + 2*b*x + 2*c*x^2])/(16*c^2) - (x*Sin[2*a + 2*b*x + 2*c*x^2])/(8*c)

Rule 3467

Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]

Rule 3464

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_))^(m_), x_Sym
bol] :> Simp[(e*(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2])/(2*c), x] + (-Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Sin[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2 \sin^2\left(a + bx + cx^2\right) dx = \int \left(\frac{x^2}{2} - \frac{1}{2}x^2 \cos\left(2a + 2bx + 2cx^2\right)\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\int x^2 \cos\left(2a + 2bx + 2cx^2\right) dx$$

$$= \frac{x^3}{6} - \frac{x \sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{\int \sin\left(2a + 2bx + 2cx^2\right) dx}{8c} + \frac{b \int x \cos\left(2a + 2bx + 2cx^2\right)}{4c}$$

$$= \frac{x^3}{6} + \frac{b \sin\left(2a + 2bx + 2cx^2\right)}{16c^2} - \frac{x \sin\left(2a + 2bx + 2cx^2\right)}{8c} - \frac{b^2 \int \cos\left(2a + 2bx + 2cx^2\right) dx}{8c^2}$$

$$= \frac{x^3}{6} + \frac{\sqrt{\pi} \cos\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{3/2}} + \frac{\sqrt{\pi} C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right) \sin\left(2a - \frac{b^2}{2c}\right)}{16c^{3/2}} + \frac{b \sin\left(2a + 2bx + 2cx^2\right)}{16c^2}$$

$$= \frac{x^3}{6} - \frac{b^2 \sqrt{\pi} \cos\left(2a - \frac{b^2}{2c}\right) C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi} \cos\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{3/2}} + \frac{\sqrt{\pi} C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right) \sin\left(2a\right.}{16c^{3/2}}$$

Mathematica [A]   time = 0.65494, size = 170, normalized size = 0.69

$$\frac{-3\sqrt{\pi}\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)\left(b^2 \cos\left(2a - \frac{b^2}{2c}\right) - c \sin\left(2a - \frac{b^2}{2c}\right)\right) + 3\sqrt{\pi}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)\left(b^2 \sin\left(2a - \frac{b^2}{2c}\right) + c \cos\left(2a - \frac{b^2}{2c}\right)\right) + \sqrt{c}\left(3\right.}{48c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (3*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*(c*Cos[2*a - b^2/(2*c)
] + b^2*Sin[2*a - b^2/(2*c)]) - 3*Sqrt[Pi]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sq
rt[Pi])]*(b^2*Cos[2*a - b^2/(2*c)] - c*Sin[2*a - b^2/(2*c)]) + Sqrt[c]*(8*c
^2*x^3 + 3*(b - 2*c*x)*Sin[2*(a + x*(b + c*x))]))/(48*c^(5/2))
```

Maple [A]   time = 0.018, size = 191, normalized size = 0.8

$$\frac{x^3}{6} - \frac{x \sin\left(2cx^2 + 2bx + 2a\right)}{8c} + \frac{b}{4c}\left(\frac{\sin\left(2cx^2 + 2bx + 2a\right)}{4c} - \frac{b\sqrt{\pi}}{4}\left(\cos\left(\frac{-4ca + b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/6*x^3-1/8*x*sin(2*c*x^2+2*b*x+2*a)/c+1/4*b/c*(1/4*sin(2*c*x^2+2*b*x+2*a)/
c-1/4*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/2
)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*FresnelS((2*c*x+b)/c^(1/2)/Pi^(1/2))))+
1/16/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a*c+b^2)/c)*FresnelS((2*c*x+b)/c^(1/2)/P
i^(1/2))-sin(1/2*(-4*a*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2)))
```

Maxima [C]   time = 3.61573, size = 4236, normalized size = 17.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*sqrt(2)*(2*sqrt(2)*(8*c^3*x^3*abs(c) - b*c*(3*I*e^(1/2*(4*I*c^2*x^2 +
 4*I*b*c*x + I*b^2)/c) - 3*I*e^(-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*
abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) + 3*b*c*(e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x +
 I*b^2)/c) + e^(-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*
(b^2 - 4*a*c)/c))*((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c))^(3/2) + (6*b^3*(gamm
a(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*
x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) - b^3*(6*I*ga
mma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 6*I*gamma(3/2, -1/2*(4*
I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c) + (48*c
^3*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(
4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) - c^3
*(48*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 48*I*gamma(3/2
, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/
c))*x^3 + (72*b*c^2*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) +
gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2
- 4*a*c)/c) - b*c^2*(72*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/
c) - 72*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(
-1/2*(b^2 - 4*a*c)/c))*x^2 + (36*b^2*c*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b
*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*ab
s(c)*cos(-1/2*(b^2 - 4*a*c)/c) - b^2*c*(36*I*gamma(3/2, 1/2*(4*I*c^2*x^2 +
4*I*b*c*x + I*b^2)/c) - 36*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b
^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))*x)*cos(3/2*arctan2(2*(4*c^2*x^2 +
 4*b*c*x + b^2)/c, 0)) - (3*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(-1/2*(b^2 - 4*a*c)/c) + (-3*I*sqrt(pi)*(
erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 3*I*sqrt(pi
)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(
-1/2*(b^2 - 4*a*c)/c) + (24*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*
```

```
I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/2*(b^2 - 4*a*c)/c) + (-24*I*sqrt(
pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 24*I*s
qrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^
2*c^3*sin(-1/2*(b^2 - 4*a*c)/c))*x^3 + (36*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/2*(b^2 - 4*a*c)/c)
 + (-36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)
) - 1) + 36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c)) - 1))*b^3*c^2*sin(-1/2*(b^2 - 4*a*c)/c))*x^2 + (18*(sqrt(pi)*(erf(sq
rt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqr
t(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(-1/2*(b^
2 - 4*a*c)/c) + (-18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*
x + I*b^2)/c)) - 1) + 18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I
*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/2*(b^2 - 4*a*c)/c))*x)*cos(1/2*arcta
n2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) - (b^3*(6*I*gamma(3/2, 1/2*(4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c) - 6*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x
 + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) + 6*b^3*(gamma(3/2, 1/2*(4*I
*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x
 + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c) + (c^3*(48*I*gamma(3/2, 1/2*
(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 48*I*gamma(3/2, -1/2*(4*I*c^2*x^2 +
4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) + 48*c^3*(gamma(3/2
, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 +
 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))*x^3 + (b*c^2*(72*
I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 72*I*gamma(3/2, -1/
2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) +
72*b*c^2*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2,
-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c)
)*x^2 + (b^2*c*(36*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) -
36*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*
(b^2 - 4*a*c)/c) + 36*b^2*c*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1
/2*(b^2 - 4*a*c)/c))*x)*sin(3/2*arctan2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)
) - ((-3*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c
)) - 1) + 3*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c)) - 1))*b^5*cos(-1/2*(b^2 - 4*a*c)/c) - 3*(sqrt(pi)*(erf(sqrt(1/2)*sqr
t((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt
(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(-1/2*(b^2 - 4*a*c)/c)
 + ((-24*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c
)) - 1) + 24*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b
^2)/c)) - 1))*b^2*c^3*cos(-1/2*(b^2 - 4*a*c)/c) - 24*(sqrt(pi)*(erf(sqrt(1/
2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2
)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*sin(-1/2*(b^2 -
 4*a*c)/c))*x^3 + ((-36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b
*c*x + I*b^2)/c)) - 1) + 36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 +
```

```
4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/2*(b^2 - 4*a*c)/c) - 36*(sqrt(p
i)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi
)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*
sin(-1/2*(b^2 - 4*a*c)/c))*x^2 + ((-18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*
c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-
(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(-1/2*(b^2 - 4*a*c)/c)
 - 18*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) -
 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) -
 1))*b^4*c*sin(-1/2*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan2(2*(4*c^2*x^2 + 4*b
*c*x + b^2)/c, 0)))/(c^3*((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c))^(3/2)*abs(c))
```

Fricas [A]   time = 1.60878, size = 440, normalized size = 1.77

$$\frac{8\,c^3x^3 - 6\left(2\,c^2x - bc\right)\cos\left(cx^2 + bx + a\right)\sin\left(cx^2 + bx + a\right) - 3\left(\pi b^2\cos\left(-\frac{b^2-4\,ac}{2\,c}\right) - \pi c\sin\left(-\frac{b^2-4\,ac}{2\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right)}{48\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*sin(c*x^2+b*x+a)^2,x, algorithm="fricas")

[Out] 1/48*(8*c^3*x^3 - 6*(2*c^2*x - b*c)*cos(c*x^2 + b*x + a)*sin(c*x^2 + b*x +
a) - 3*(pi*b^2*cos(-1/2*(b^2 - 4*a*c)/c) - pi*c*sin(-1/2*(b^2 - 4*a*c)/c))*
sqrt(c/pi)*fresnel_cos((2*c*x + b)*sqrt(c/pi)/c) + 3*(pi*b^2*sin(-1/2*(b^2
- 4*a*c)/c) + pi*c*cos(-1/2*(b^2 - 4*a*c)/c))*sqrt(c/pi)*fresnel_sin((2*c*x
 + b)*sqrt(c/pi)/c))/c^3

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin^2\left(a + bx + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*sin(c*x**2+b*x+a)**2,x)

[Out] Integral(x**2*sin(a + b*x + c*x**2)**2, x)

Giac [C]   time = 1.24306, size = 289, normalized size = 1.17

$$\frac{1}{6}\,x^3 - \frac{\left(c\left(-2i\,x - \frac{i\,b}{c}\right) + 2i\,b\right)e^{\left(2i\,cx^2+2i\,bx+2i\,a\right)} - \frac{\sqrt{\pi}\left(b^2+i\,c\right)\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2\,x+\frac{b}{c}\right)\left(-\frac{i\,c}{|c|}+1\right)\right)e^{\left(-\frac{i\,b^2-4i\,ac}{2\,c}\right)}}{\sqrt{c}\left(-\frac{i\,c}{|c|}+1\right)}}{32\,c^2} - \frac{\left(c\left(2i\,x + \frac{i\,b}{c}\right) - 2i\,b\right)e^{\left(-2i\,cx^2-2i\,bx\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/32*((c*(-2*I*x - I*b/c) + 2*I*b)*e^(2*I*c*x^2 + 2*I*b*x + 2*I*a
) - sqrt(pi)*(b^2 + I*c)*erf(-1/2*sqrt(c)*(2*x + b/c)*(-I*c/abs(c) + 1))*e^
(-1/2*(I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)))/c^2 - 1/32*((c*(2*I
*x + I*b/c) - 2*I*b)*e^(-2*I*c*x^2 - 2*I*b*x - 2*I*a) - sqrt(pi)*(b^2 - I*c
)*erf(-1/2*sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 + 4*I*a*c)
/c)/(sqrt(c)*(I*c/abs(c) + 1)))/c^2
```

3.17 $\int x \sin^2\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=126

$$\frac{\sqrt{\pi}b \cos\left(2a - \frac{b^2}{2c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{8c^{3/2}} - \frac{\sqrt{\pi}b \sin\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} - \frac{\sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{x^2}{4}$$

```
[Out] x^2/4 + (b*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt
[Pi])])/(8*c^(3/2)) - (b*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*
Sin[2*a - b^2/(2*c)])/(8*c^(3/2)) - Sin[2*a + 2*b*x + 2*c*x^2]/(8*c)
```

Rubi [A]   time = 0.0728105, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3467, 3462, 3448, 3352, 3351}

$$\frac{\sqrt{\pi}b \cos\left(2a - \frac{b^2}{2c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{8c^{3/2}} - \frac{\sqrt{\pi}b \sin\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} - \frac{\sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] x^2/4 + (b*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt
[Pi])])/(8*c^(3/2)) - (b*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*
Sin[2*a - b^2/(2*c)])/(8*c^(3/2)) - Sin[2*a + 2*b*x + 2*c*x^2]/(8*c)
```

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int x \sin^2\left(a + bx + cx^2\right) dx = \int \left(\frac{x}{2} - \frac{1}{2}x \cos\left(2a + 2bx + 2cx^2\right)\right) dx$$

$$= \frac{x^2}{4} - \frac{1}{2} \int x \cos\left(2a + 2bx + 2cx^2\right) dx$$

$$= \frac{x^2}{4} - \frac{\sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{b \int \cos\left(2a + 2bx + 2cx^2\right) dx}{4c}$$

$$= \frac{x^2}{4} - \frac{\sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{\left(b \cos\left(2a - \frac{b^2}{2c}\right)\right) \int \cos\left(\frac{(2b+4cx)^2}{8c}\right) dx}{4c} - \frac{\left(b \sin\left(2a - \frac{b^2}{2c}\right)\right) \int s}{4c}$$

$$= \frac{x^2}{4} + \frac{b\sqrt{\pi} \cos\left(2a - \frac{b^2}{2c}\right) C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} - \frac{b\sqrt{\pi} S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right) \sin\left(2a - \frac{b^2}{2c}\right)}{8c^{3/2}} - \frac{\sin\left(2a + 2bx + 2cx^2\right)}{8c}$$

Mathematica [A]   time = 0.280254, size = 118, normalized size = 0.94

$$\frac{\sqrt{\pi}b \cos\left(2a - \frac{b^2}{2c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right) - \sqrt{\pi}b \sin\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right) + \sqrt{c}\left(2cx^2 - \sin(2(a + x(b + cx)))\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (b*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])] -
 b*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)] +
```

```
Sqrt[c]*(2*c*x^2 - Sin[2*(a + x*(b + c*x))])]))/(8*c^(3/2))
```

Maple [A]   time = 0.016, size = 95, normalized size = 0.8

$$\frac{x^2}{4} - \frac{\sin\left(2\,cx^2 + 2\,bx + 2\,a\right)}{8\,c} + \frac{b\sqrt{\pi}}{8}\left(\cos\left(\frac{-4\,ca + b^2}{2\,c}\right)\text{FresnelC}\left(\frac{2\,cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{-4\,ca + b^2}{2\,c}\right)\text{FresnelS}\left(\frac{2\,cx + b}{\sqrt{\pi}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/4*x^2-1/8*sin(2*c*x^2+2*b*x+2*a)/c+1/8*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a*
c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*Fres
nelS((2*c*x+b)/c^(1/2)/Pi^(1/2)))
```

Maxima [C]   time = 2.1199, size = 1372, normalized size = 10.89

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*(sqrt(2)*(4*c^2*x^2 + c*(I*e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I
*b^2)/c) - I*e^(-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*cos(-1/2*(b^2 -
4*a*c)/c) - c*(e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + e^(-1/2*(4*I*c
^2*x^2 + 4*I*b*c*x + I*b^2)/c))*sin(-1/2*(b^2 - 4*a*c)/c))*sqrt((4*c^2*x^2
+ 4*b*c*x + b^2)/abs(c)) + ((sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(-1/2*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(er
f(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt(pi)*(e
rf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/2
*(b^2 - 4*a*c)/c) + (2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c
*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c
*x + I*b^2)/c)) - 1))*b*c*cos(-1/2*(b^2 - 4*a*c)/c) + (-2*I*sqrt(pi)*(erf(s
qrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(er
f(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/2*
(b^2 - 4*a*c)/c))*x)*cos(1/2*arctan2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) +
 ((-I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) -
```

```
1) + I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
- 1))*b^2*cos(-1/2*(b^2 - 4*a*c)/c) - (sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/2*(b^2 - 4*a*c)/c) + ((-2*I
*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) +
2*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1
))*b*c*cos(-1/2*(b^2 - 4*a*c)/c) - 2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/2*(b^2 - 4*a*c)/c))*x)*sin(1
/2*arctan2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)))/(c^2*sqrt((4*c^2*x^2 + 4*b
*c*x + b^2)/abs(c)))
```

Fricas [A]   time = 1.63334, size = 315, normalized size = 2.5

$$\frac{\pi b \sqrt{\frac{c}{\pi}} \cos\left(-\frac{b^2-4\,ac}{2\,c}\right) \mathrm{C}\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right) - \pi b \sqrt{\frac{c}{\pi}} \mathrm{S}\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right) \sin\left(-\frac{b^2-4\,ac}{2\,c}\right) + 2\,c^2 x^2 - 2\,c\cos\left(cx^2+bx+a\right)\sin\left(cx^2+bx+\right.}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(pi*b*sqrt(c/pi)*cos(-1/2*(b^2 - 4*a*c)/c)*fresnel_cos((2*c*x + b)*sqrt
(c/pi)/c) - pi*b*sqrt(c/pi)*fresnel_sin((2*c*x + b)*sqrt(c/pi)/c)*sin(-1/2*
(b^2 - 4*a*c)/c) + 2*c^2*x^2 - 2*c*cos(c*x^2 + b*x + a)*sin(c*x^2 + b*x + a
))/c^2
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x \sin^2\left(a + bx + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(c*x**2+b*x+a)**2,x)
```

```
[Out] Integral(x*sin(a + b*x + c*x**2)**2, x)
```

Giac [C]   time = 1.24837, size = 230, normalized size = 1.83

$$\frac{1}{4}x^2 - \frac{\dfrac{\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\right)e^{\left(-\frac{ib^2-4iac}{2c}\right)}}{\sqrt{c}\left(-\frac{ic}{|c|}+1\right)} - ie^{\left(2icx^2+2ibx+2ia\right)}}{16\,c} - \frac{\dfrac{\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\right)e^{\left(-\frac{-ib^2+4iac}{2c}\right)}}{\sqrt{c}\left(\frac{ic}{|c|}+1\right)} + ie^{\left(-2icx^2-2ibx-2ia\right)}}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/16*(sqrt(pi)*b*erf(-1/2*sqrt(c)*(2*x + b/c)*(-I*c/abs(c) + 1))*
e^(-1/2*(I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) - I*e^(2*I*c*x^2 +
 2*I*b*x + 2*I*a))/c - 1/16*(sqrt(pi)*b*erf(-1/2*sqrt(c)*(2*x + b/c)*(I*c/a
bs(c) + 1))*e^(-1/2*(-I*b^2 + 4*I*a*c)/c)/(sqrt(c)*(I*c/abs(c) + 1)) + I*e^
(-2*I*c*x^2 - 2*I*b*x - 2*I*a))/c
```

3.18 $\int \sin^2\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)\operatorname{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{x}{2}$$

```
[Out] x/2 - (Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi]
)])/(4*Sqrt[c]) + (Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*
a - b^2/(2*c)])/(4*Sqrt[c])
```

Rubi [A]   time = 0.0495308, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3449, 3448, 3352, 3351}

$$-\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)\operatorname{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] x/2 - (Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi]
)])/(4*Sqrt[c]) + (Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*
a - b^2/(2*c)])/(4*Sqrt[c])
```

Rule 3449

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_), x_Symbol] :> Int[ExpandTri
gReduce[Sin[a + b*x + c*x^2]^n, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 1]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin^2\left(a + bx + cx^2\right) dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos\left(2a + 2bx + 2cx^2\right)\right) dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos\left(2a + 2bx + 2cx^2\right) dx$$

$$= \frac{x}{2} - \frac{1}{2}\cos\left(2a - \frac{b^2}{2c}\right)\int \cos\left(\frac{(2b + 4cx)^2}{8c}\right) dx + \frac{1}{2}\sin\left(2a - \frac{b^2}{2c}\right)\int \sin\left(\frac{(2b + 4cx)^2}{8c}\right) dx$$

$$= \frac{x}{2} - \frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)\sin\left(2a - \frac{b^2}{2c}\right)}{4\sqrt{c}}$$

Mathematica [A]   time = 0.103324, size = 97, normalized size = 0.97

$$\frac{-\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right) + \sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right) + 2\sqrt{c}x}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (2*Sqrt[c]*x - Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*
Sqrt[Pi])] + Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^
2/(2*c)])/(4*Sqrt[c])
```

Maple [A]   time = 0.013, size = 72, normalized size = 0.7

$$\frac{x}{2} - \frac{\sqrt{\pi}}{4}\left(\cos\left(\frac{-4ca + b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{-4ca + b^2}{2c}\right)\text{FresnelS}\left(\frac{2cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right)\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/2*x-1/4*Pi^(1/2)/c^(1/2)*(cos(1/2*(-4*a*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1
/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*FresnelS((2*c*x+b)/c^(1/2)/Pi^(1/2)))
```

Maxima [C]   time = 1.88765, size = 410, normalized size = 4.1

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)-i\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+i\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(2)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) - (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)
) - sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(2*I*c)) - ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) + (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)
) + sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(-2*I*c)))*sqrt(abs(c)) - 16*x*abs(c))/abs(c)
```

Fricas [A]   time = 1.5353, size = 238, normalized size = 2.38

$$-\frac{\pi\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{2\,c}\right)C\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right)-\pi\sqrt{\frac{c}{\pi}}S\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right)\sin\left(-\frac{b^2-4\,ac}{2\,c}\right)-2\,cx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(pi*sqrt(c/pi)*cos(-1/2*(b^2 - 4*a*c)/c)*fresnel_cos((2*c*x + b)*sqrt(
c/pi)/c) - pi*sqrt(c/pi)*fresnel_sin((2*c*x + b)*sqrt(c/pi)/c)*sin(-1/2*(b^
2 - 4*a*c)/c) - 2*c*x)/c
```

Sympy [A]   time = 1.1142, size = 83, normalized size = 0.83

$$\frac{x}{2} - \frac{\sqrt{\pi}\left(-\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{2b+4cx}{2\sqrt{\pi}\sqrt{c}}\right) + \cos\left(2a - \frac{b^2}{2c}\right)C\left(\frac{2b+4cx}{2\sqrt{\pi}\sqrt{c}}\right)\right)\sqrt{\frac{1}{c}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x**2+b*x+a)**2,x)
```

```
[Out] x/2 - sqrt(pi)*(-sin(2*a - b**2/(2*c))*fresnels((2*b + 4*c*x)/(2*sqrt(pi)*s
qrt(c))) + cos(2*a - b**2/(2*c))*fresnelc((2*b + 4*c*x)/(2*sqrt(pi)*sqrt(c)
)))*sqrt(1/c)/4
```

Giac [C]   time = 1.27843, size = 165, normalized size = 1.65

$$\frac{1}{2}x + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x + \frac{b}{c}\right)\left(-\frac{ic}{|c|} + 1\right)\right)e^{\left(-\frac{ib^2-4iac}{2c}\right)}}{8\sqrt{c}\left(-\frac{ic}{|c|} + 1\right)} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x + \frac{b}{c}\right)\left(\frac{ic}{|c|} + 1\right)\right)e^{\left(-\frac{-ib^2+4iac}{2c}\right)}}{8\sqrt{c}\left(\frac{ic}{|c|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/8*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + b/c)*(-I*c/abs(c) + 1))*e^(-1/
2*(I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) + 1/8*sqrt(pi)*erf(-1/2*
sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 + 4*I*a*c)/c)/(sqrt(c
)*(I*c/abs(c) + 1))
```

3.19 $\int \dfrac{\sin^2\left(a+bx+cx^2\right)}{x}\,dx$

Optimal. Leaf size=32

$$\frac{\log(x)}{2} - \frac{1}{2}\text{Unintegrable}\left(\frac{\cos\left(2a+2bx+2cx^2\right)}{x}, x\right)$$

[Out] Log[x]/2 - Unintegrable[Cos[2*a + 2*b*x + 2*c*x^2]/x, x]/2

Rubi [A]   time = 0.0268694, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2\left(a+bx+cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[a + b*x + c*x^2]^2/x,x]

[Out] Log[x]/2 - Defer[Int][Cos[2*a + 2*b*x + 2*c*x^2]/x, x]/2

Rubi steps

$$\int \frac{\sin^2\left(a+bx+cx^2\right)}{x}\,dx = \int \left(\frac{1}{2x} - \frac{\cos\left(2a+2bx+2cx^2\right)}{2x}\right)dx$$

$$= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos\left(2a+2bx+2cx^2\right)}{x}\,dx$$

Mathematica [A]   time = 6.99225, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a+bx+cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + b*x + c*x^2]^2/x,x]

```
[Out] Integrate[Sin[a + b*x + c*x^2]^2/x, x]
```

Maple [A]   time = 0.099, size = 0, normalized size = 0.

$$\int \frac{\left(\sin \left(cx^2 + bx + a \right) \right)^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*x^2+b*x+a)^2/x,x)
```

```
[Out] int(sin(c*x^2+b*x+a)^2/x,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \int \frac{\cos \left(2\,cx^2 + 2\,bx + 2\,a \right)}{x} \, dx + \frac{1}{2} \, \log \left(x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*integrate(cos(2*c*x^2 + 2*b*x + 2*a)/x, x) + 1/2*log(x)
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos \left(cx^2 + bx + a \right)^2 - 1}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(cos(c*x^2 + b*x + a)^2 - 1)/x, x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a + bx + cx^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(c*x**2+b*x+a)**2/x,x)

[Out] Integral(sin(a + b*x + c*x**2)**2/x, x)

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(c*x^2+b*x+a)^2/x,x, algorithm="giac")

[Out] integrate(sin(c*x^2 + b*x + a)^2/x, x)

3.20 $\quad \int x^2 \sin^2\left(a + bx - cx^2\right) dx$

Optimal. Leaf size=248

$$\frac{\sqrt{\pi}\sin\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{3/2}} + \frac{\sqrt{\pi}b^2\cos\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}b^2\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} - \sqrt{\pi}\cos$$

```
[Out] x^3/6 + (b^2*Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sq
rt[Pi]])])/(16*c^(5/2)) - (Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelS[(b - 2*c*x
)/(Sqrt[c]*Sqrt[Pi]])])/(16*c^(3/2)) + (Sqrt[Pi]*FresnelC[(b - 2*c*x)/(Sqrt[
c]*Sqrt[Pi]])*Sin[2*a + b^2/(2*c)]])/(16*c^(3/2)) + (b^2*Sqrt[Pi]*FresnelS[(
b - 2*c*x)/(Sqrt[c]*Sqrt[Pi]])*Sin[2*a + b^2/(2*c)]])/(16*c^(5/2)) + (b*Sin[
2*a + 2*b*x - 2*c*x^2])/(16*c^2) + (x*Sin[2*a + 2*b*x - 2*c*x^2])/(8*c)
```

Rubi [A] $\quad$ time = 0.219618, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3467, 3464, 3447, 3351, 3352, 3462, 3448}

$$\frac{\sqrt{\pi}\sin\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{3/2}} + \frac{\sqrt{\pi}b^2\cos\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}b^2\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} - \sqrt{\pi}\cos$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x - c*x^2]^2,x]
```

```
[Out] x^3/6 + (b^2*Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sq
rt[Pi]])])/(16*c^(5/2)) - (Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelS[(b - 2*c*x
)/(Sqrt[c]*Sqrt[Pi]])])/(16*c^(3/2)) + (Sqrt[Pi]*FresnelC[(b - 2*c*x)/(Sqrt[
c]*Sqrt[Pi]])*Sin[2*a + b^2/(2*c)]])/(16*c^(3/2)) + (b^2*Sqrt[Pi]*FresnelS[(
b - 2*c*x)/(Sqrt[c]*Sqrt[Pi]])*Sin[2*a + b^2/(2*c)]])/(16*c^(5/2)) + (b*Sin[
2*a + 2*b*x - 2*c*x^2])/(16*c^2) + (x*Sin[2*a + 2*b*x - 2*c*x^2])/(8*c)
```

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3464

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_))^(m_), x_Sym
bol] :> Simp[(e*(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2])/(2*c), x] + (-Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Sin[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2 \sin^2\left(a + bx - cx^2\right) dx = \int \left(\frac{x^2}{2} - \frac{1}{2}x^2 \cos\left(2a + 2bx - 2cx^2\right)\right) dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\int x^2 \cos\left(2a + 2bx - 2cx^2\right) dx$$

$$= \frac{x^3}{6} + \frac{x \sin\left(2a + 2bx - 2cx^2\right)}{8c} - \frac{\int \sin\left(2a + 2bx - 2cx^2\right) dx}{8c} - \frac{b \int x \cos\left(2a + 2bx - 2cx^2\right)}{4c}$$

$$= \frac{x^3}{6} + \frac{b \sin\left(2a + 2bx - 2cx^2\right)}{16c^2} + \frac{x \sin\left(2a + 2bx - 2cx^2\right)}{8c} - \frac{b^2 \int \cos\left(2a + 2bx - 2cx^2\right) dx}{8c^2}$$

$$= \frac{x^3}{6} - \frac{\sqrt{\pi} \cos\left(2a + \frac{b^2}{2c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{3/2}} + \frac{\sqrt{\pi} C\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right) \sin\left(2a + \frac{b^2}{2c}\right)}{16c^{3/2}} + \frac{b \sin\left(2a + 2bx - 2cx^2\right)}{16c^2}$$

$$= \frac{x^3}{6} + \frac{b^2\sqrt{\pi} \cos\left(2a + \frac{b^2}{2c}\right) C\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} - \frac{\sqrt{\pi} \cos\left(2a + \frac{b^2}{2c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{3/2}} + \frac{\sqrt{\pi} C\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right) \sin\left(2a\right)}{16c^{3/2}}$$

Mathematica [A]   time = 0.730072, size = 175, normalized size = 0.71

$$\frac{-3\sqrt{\pi}\text{FresnelC}\left(\frac{2cx-b}{\sqrt{\pi}\sqrt{c}}\right)\left(c \sin\left(2a + \frac{b^2}{2c}\right) + b^2 \cos\left(2a + \frac{b^2}{2c}\right)\right) + 3\sqrt{\pi}S\left(\frac{2cx-b}{\sqrt{c}\sqrt{\pi}}\right)\left(c \cos\left(2a + \frac{b^2}{2c}\right) - b^2 \sin\left(2a + \frac{b^2}{2c}\right)\right) + \sqrt{c}\,\bigg(}{48c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x - c*x^2]^2,x]
```

```
[Out] (3*Sqrt[Pi]*FresnelS[(-b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*(c*Cos[2*a + b^2/(2*c
)] - b^2*Sin[2*a + b^2/(2*c)]) - 3*Sqrt[Pi]*FresnelC[(-b + 2*c*x)/(Sqrt[c]*
Sqrt[Pi])]*(b^2*Cos[2*a + b^2/(2*c)] + c*Sin[2*a + b^2/(2*c)]) + Sqrt[c]*(8
*c^2*x^3 + 3*(b + 2*c*x)*Sin[2*(a + x*(b - c*x))]))/(48*c^(5/2))
```

Maple [A]   time = 0.017, size = 199, normalized size = 0.8

$$\frac{x^3}{6} + \frac{x \sin\left(-2cx^2 + 2bx + 2a\right)}{8c} - \frac{b}{4c}\left(-\frac{\sin\left(-2cx^2 + 2bx + 2a\right)}{4c} + \frac{b\sqrt{\pi}}{4}\left(\cos\left(\frac{4ca + b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx-b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + s\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(-c*x^2+b*x+a)^2,x)
```

```
[Out] 1/6*x^3+1/8*x*sin(-2*c*x^2+2*b*x+2*a)/c-1/4*b/c*(-1/4*sin(-2*c*x^2+2*
a)/c+1/4*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(4*a*c+b^2)/c)*FresnelC(1/Pi^(1/2)/c^(
1/2)*(2*c*x-b))+sin(1/2*(4*a*c+b^2)/c)*FresnelS(1/Pi^(1/2)/c^(1/2)*(2*c*x-b
))))+1/16/c^(3/2)*Pi^(1/2)*(cos(1/2*(4*a*c+b^2)/c)*FresnelS(1/Pi^(1/2)/c^(1
/2)*(2*c*x-b))-sin(1/2*(4*a*c+b^2)/c)*FresnelC(1/Pi^(1/2)/c^(1/2)*(2*c*x-b)
))
```

Maxima [C]   time = 2.20835, size = 4238, normalized size = 17.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(-c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*sqrt(2)*(2*sqrt(2)*(8*c^3*x^3*abs(c) + b*c*(3*I*e^(1/2*(4*I*c^2*x^2 -
 4*I*b*c*x + I*b^2)/c) - 3*I*e^(-1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*
abs(c)*cos(1/2*(b^2 + 4*a*c)/c) + 3*b*c*(e^(1/2*(4*I*c^2*x^2 - 4*I*b*c*x +
I*b^2)/c) + e^(-1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/2*(b
^2 + 4*a*c)/c))*((4*c^2*x^2 - 4*b*c*x + b^2)/abs(c))^(3/2) - (6*b^3*(gamma(
3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^
2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/2*(b^2 + 4*a*c)/c) - b^3*(-6*I*gamm
a(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + 6*I*gamma(3/2, -1/2*(4*I*
c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/2*(b^2 + 4*a*c)/c) - (48*c^3*
(gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I
*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/2*(b^2 + 4*a*c)/c) + c^3*(48
*I*gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) - 48*I*gamma(3/2, -1
/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/2*(b^2 + 4*a*c)/c))*x
^3 + (72*b*c^2*(gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma
(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/2*(b^2 + 4*a*
c)/c) - b*c^2*(-72*I*gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) +
72*I*gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/2*(
b^2 + 4*a*c)/c))*x^2 - (36*b^2*c*(gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x +
 I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*c
os(1/2*(b^2 + 4*a*c)/c) + b^2*c*(36*I*gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c
*x + I*b^2)/c) - 36*I*gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))
*abs(c)*sin(1/2*(b^2 + 4*a*c)/c))*x)*cos(3/2*arctan2(2*(4*c^2*x^2 - 4*b*c*x
 + b^2)/c, 0)) + (3*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x
+ I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x
+ I*b^2)/c)) - 1))*b^5*cos(1/2*(b^2 + 4*a*c)/c) + (3*I*sqrt(pi)*(erf(sqrt(1
/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - 3*I*sqrt(pi)*(erf(sqr
t(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(1/2*(b^2 +
```

```
4*a*c)/c) - (24*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I
*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I
*b^2)/c)) - 1))*b^2*c^3*cos(1/2*(b^2 + 4*a*c)/c) - (-24*I*sqrt(pi)*(erf(sqr
t(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 24*I*sqrt(pi)*(erf
(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*sin(1/
2*(b^2 + 4*a*c)/c))*x^3 + (36*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 -
4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 -
4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(1/2*(b^2 + 4*a*c)/c) + (36*I*sqrt(
pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - 36*I*s
qrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^
3*c^2*sin(1/2*(b^2 + 4*a*c)/c))*x^2 - (18*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*
I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*
I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(1/2*(b^2 + 4*a*c)/c) - (
-18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) -
1) + 18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c
)) - 1))*b^4*c*sin(1/2*(b^2 + 4*a*c)/c))*x)*cos(1/2*arctan2(2*(4*c^2*x^2 -
4*b*c*x + b^2)/c, 0)) + (b^3*(6*I*gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x +
 I*b^2)/c) - 6*I*gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(
c)*cos(1/2*(b^2 + 4*a*c)/c) - 6*b^3*(gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*
x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c
)*sin(1/2*(b^2 + 4*a*c)/c) + (c^3*(-48*I*gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c) + 48*I*gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/
c))*abs(c)*cos(1/2*(b^2 + 4*a*c)/c) + 48*c^3*(gamma(3/2, 1/2*(4*I*c^2*x^2 -
 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/
c))*abs(c)*sin(1/2*(b^2 + 4*a*c)/c))*x^3 + (b*c^2*(72*I*gamma(3/2, 1/2*(4*I
*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) - 72*I*gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c))*abs(c)*cos(1/2*(b^2 + 4*a*c)/c) - 72*b*c^2*(gamma(3/2, 1
/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 - 4*
I*b*c*x + I*b^2)/c))*abs(c)*sin(1/2*(b^2 + 4*a*c)/c))*x^2 + (b^2*c*(-36*I*g
amma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + 36*I*gamma(3/2, -1/2*(
4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(1/2*(b^2 + 4*a*c)/c) + 36*b
^2*c*(gamma(3/2, 1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2
*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(1/2*(b^2 + 4*a*c)/c))*x)*
sin(3/2*arctan2(2*(4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)) + ((-3*I*sqrt(pi)*(erf
(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 3*I*sqrt(pi)*(
erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(1/2
*(b^2 + 4*a*c)/c) + 3*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*
x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*
x + I*b^2)/c)) - 1))*b^5*sin(1/2*(b^2 + 4*a*c)/c) + ((24*I*sqrt(pi)*(erf(sq
rt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - 24*I*sqrt(pi)*(er
f(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(1
/2*(b^2 + 4*a*c)/c) - 24*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b
*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b
*c*x + I*b^2)/c)) - 1))*b^2*c^3*sin(1/2*(b^2 + 4*a*c)/c))*x^3 + ((-36*I*sqr
t(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 36*I
```

```
*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*
b^3*c^2*cos(1/2*(b^2 + 4*a*c)/c) + 36*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^
2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^
2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(1/2*(b^2 + 4*a*c)/c))*x^2
+ ((18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))
 - 1) - 18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2
)/c)) - 1))*b^4*c*cos(1/2*(b^2 + 4*a*c)/c) - 18*(sqrt(pi)*(erf(sqrt(1/2)*sq
rt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqr
t(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(1/2*(b^2 + 4*a*c)/
c))*x)*sin(1/2*arctan2(2*(4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)))/(c^3*((4*c^2*x
^2 - 4*b*c*x + b^2)/abs(c))^(3/2)*abs(c))
```

Fricas [A]   time = 1.49675, size = 435, normalized size = 1.75

$$\frac{8\,c^3x^3 - 6\left(2\,c^2x + bc\right)\cos\left(cx^2 - bx - a\right)\sin\left(cx^2 - bx - a\right) - 3\left(\pi b^2\cos\left(\frac{b^2+4\,ac}{2\,c}\right) + \pi c\sin\left(\frac{b^2+4\,ac}{2\,c}\right)\right)\sqrt{\frac{c}{\pi}}\,C\left(\frac{(2\,cx-b)\sqrt{\frac{c}{\pi}}}{c}\right) - \dots}{48\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(-c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(8*c^3*x^3 - 6*(2*c^2*x + b*c)*cos(c*x^2 - b*x - a)*sin(c*x^2 - b*x -
a) - 3*(pi*b^2*cos(1/2*(b^2 + 4*a*c)/c) + pi*c*sin(1/2*(b^2 + 4*a*c)/c))*sq
rt(c/pi)*fresnel_cos((2*c*x - b)*sqrt(c/pi)/c) - 3*(pi*b^2*sin(1/2*(b^2 + 4
*a*c)/c) - pi*c*cos(1/2*(b^2 + 4*a*c)/c))*sqrt(c/pi)*fresnel_sin((2*c*x - b
)*sqrt(c/pi)/c))/c^3
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x^2\sin^2\left(a + bx - cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(-c*x**2+b*x+a)**2,x)
```

```
[Out] Integral(x**2*sin(a + b*x - c*x**2)**2, x)
```

Giac [C]   time = 1.28688, size = 292, normalized size = 1.18

$$\frac{1}{6}\,x^3 - \frac{\left(c\left(-2i\,x + \frac{i\,b}{c}\right) - 2i\,b\right)e^{\left(2i\,cx^2 - 2i\,bx - 2i\,a\right)} - \dfrac{\sqrt{\pi}\left(b^2 + i\,c\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x - \frac{b}{c}\right)\left(-\frac{i\,c}{|c|} + 1\right)\right)e^{\left(-\frac{i\,b^2 + 4i\,ac}{2\,c}\right)}}{\sqrt{c}\left(-\frac{i\,c}{|c|} + 1\right)}}{32\,c^2} - \frac{\left(c\left(2i\,x - \frac{i\,b}{c}\right) + 2i\,b\right)e^{\left(-2i\,cx^2 + 2i\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(-c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/32*((c*(-2*I*x + I*b/c) - 2*I*b)*e^(2*I*c*x^2 - 2*I*b*x - 2*I*a
) - sqrt(pi)*(b^2 + I*c)*erf(-1/2*sqrt(c)*(2*x - b/c)*(-I*c/abs(c) + 1))*e^
(-1/2*(I*b^2 + 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)))/c^2 - 1/32*((c*(2*I
*x - I*b/c) + 2*I*b)*e^(-2*I*c*x^2 + 2*I*b*x + 2*I*a) - sqrt(pi)*(b^2 - I*c
)*erf(-1/2*sqrt(c)*(2*x - b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 - 4*I*a*c)
/c)/(sqrt(c)*(I*c/abs(c) + 1)))/c^2
```

3.21 $\int x \sin^2\left(a + bx - cx^2\right) dx$

Optimal. Leaf size=126

$$\frac{\sqrt{\pi} b \cos\left(2a + \frac{b^2}{2c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{8c^{3/2}} + \frac{\sqrt{\pi} b \sin\left(2a + \frac{b^2}{2c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} + \frac{\sin\left(2a + 2bx - 2cx^2\right)}{8c} + \frac{x^2}{4}$$

```
[Out] x^2/4 + (b*Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt
[Pi])])/(8*c^(3/2)) + (b*Sqrt[Pi]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*
Sin[2*a + b^2/(2*c)])/(8*c^(3/2)) + Sin[2*a + 2*b*x - 2*c*x^2]/(8*c)
```

Rubi [A]   time = 0.0663787, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3467, 3462, 3448, 3352, 3351}

$$\frac{\sqrt{\pi} b \cos\left(2a + \frac{b^2}{2c}\right) \text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{8c^{3/2}} + \frac{\sqrt{\pi} b \sin\left(2a + \frac{b^2}{2c}\right) S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} + \frac{\sin\left(2a + 2bx - 2cx^2\right)}{8c} + \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + b*x - c*x^2]^2,x]
```

```
[Out] x^2/4 + (b*Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt
[Pi])])/(8*c^(3/2)) + (b*Sqrt[Pi]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*
Sin[2*a + b^2/(2*c)])/(8*c^(3/2)) + Sin[2*a + 2*b*x - 2*c*x^2]/(8*c)
```

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\int x \sin^2\left(a + bx - cx^2\right) dx = \int \left(\frac{x}{2} - \frac{1}{2}x\cos\left(2a + 2bx - 2cx^2\right)\right) dx
$$

$$
= \frac{x^2}{4} - \frac{1}{2}\int x\cos\left(2a + 2bx - 2cx^2\right) dx
$$

$$
= \frac{x^2}{4} + \frac{\sin\left(2a + 2bx - 2cx^2\right)}{8c} - \frac{b\int \cos\left(2a + 2bx - 2cx^2\right) dx}{4c}
$$

$$
= \frac{x^2}{4} + \frac{\sin\left(2a + 2bx - 2cx^2\right)}{8c} - \frac{\left(b\cos\left(2a + \frac{b^2}{2c}\right)\right)\int \cos\left(\frac{(2b-4cx)^2}{8c}\right) dx}{4c} - \frac{\left(b\sin\left(2a + \frac{b^2}{2c}\right)\right)\int}{4}
$$

$$
= \frac{x^2}{4} + \frac{b\sqrt{\pi}\cos\left(2a + \frac{b^2}{2c}\right)C\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} + \frac{b\sqrt{\pi}S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)\sin\left(2a + \frac{b^2}{2c}\right)}{8c^{3/2}} + \frac{\sin\left(2a + 2bx - 2cx^2\right)}{8c}
$$

Mathematica [A]   time = 0.291463, size = 122, normalized size = 0.97

$$
\frac{\sqrt{\pi}(-b)\cos\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx-b}{\sqrt{\pi}\sqrt{c}}\right) - \sqrt{\pi}b\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{2cx-b}{\sqrt{c}\sqrt{\pi}}\right) + \sqrt{c}\left(\sin(2(a + x(b - cx))) + 2cx^2\right)}{8c^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + b*x - c*x^2]^2,x]
```

```
[Out] (-(b*Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(-b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])
]) - b*Sqrt[Pi]*FresnelS[(-b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a + b^2/(2*
```

```
c)] + Sqrt[c]*(2*c*x^2 + Sin[2*(a + x*(b - c*x))])]))/(8*c^(3/2))
```

Maple [A]   time = 0.014, size = 99, normalized size = 0.8

$$\frac{x^2}{4} + \frac{\sin\left(-2\,cx^2 + 2\,bx + 2\,a\right)}{8\,c} - \frac{b\sqrt{\pi}}{8}\left(\cos\left(\frac{4\,ca + b^2}{2\,c}\right)\mathrm{FresnelC}\left(\frac{2\,cx - b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{4\,ca + b^2}{2\,c}\right)\mathrm{FresnelS}\left(\frac{2\,cx - b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*sin(-c*x^2+b*x+a)^2,x)
```

```
[Out] 1/4*x^2+1/8*sin(-2*c*x^2+2*b*x+2*a)/c-1/8*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(4*a*
c+b^2)/c)*FresnelC(1/Pi^(1/2)/c^(1/2)*(2*c*x-b))+sin(1/2*(4*a*c+b^2)/c)*Fre
snelS(1/Pi^(1/2)/c^(1/2)*(2*c*x-b)))
```

Maxima [C]   time = 1.95154, size = 1372, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*sin(-c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*(sqrt(2)*(4*c^2*x^2 + c*(I*e^(1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I
*b^2)/c) - I*e^(-1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c))*cos(1/2*(b^2 + 4
*a*c)/c) + c*(e^(1/2*(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c) + e^(-1/2*(4*I*c^
2*x^2 - 4*I*b*c*x + I*b^2)/c))*sin(1/2*(b^2 + 4*a*c)/c))*sqrt((4*c^2*x^2 -
4*b*c*x + b^2)/abs(c)) + ((sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c)) - 1))*b^2*cos(1/2*(b^2 + 4*a*c)/c) + (I*sqrt(pi)*(erf(sq
rt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - I*sqrt(pi)*(erf(s
qrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(1/2*(b^2
 + 4*a*c)/c) - (2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x +
I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x +
I*b^2)/c)) - 1))*b*c*cos(1/2*(b^2 + 4*a*c)/c) - (-2*I*sqrt(pi)*(erf(sqrt(1/
2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(erf(sqrt
(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(1/2*(b^2 +
4*a*c)/c))*x)*cos(1/2*arctan2(2*(4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)) + ((-I*s
qrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) + I*
```

```
sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b
^2*cos(1/2*(b^2 + 4*a*c)/c) + (sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 -
4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 -
4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(1/2*(b^2 + 4*a*c)/c) + ((2*I*sqrt(pi)*
(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1) - 2*I*sqrt(p
i)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*cos
(1/2*(b^2 + 4*a*c)/c) - 2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 - 4*I*
b*c*x + I*b^2)/c)) - 1))*b*c*sin(1/2*(b^2 + 4*a*c)/c))*x)*sin(1/2*arctan2(2
*(4*c^2*x^2 - 4*b*c*x + b^2)/c, 0)))/(c^2*sqrt((4*c^2*x^2 - 4*b*c*x + b^2)/
abs(c)))
```

Fricas [A]   time = 1.4373, size = 313, normalized size = 2.48

$$-\frac{\pi b\sqrt{\frac{c}{\pi}}\cos\left(\frac{b^2+4\,ac}{2\,c}\right)\mathrm{C}\left(\frac{(2\,cx-b)\sqrt{\frac{c}{\pi}}}{c}\right) + \pi b\sqrt{\frac{c}{\pi}}\,\mathrm{S}\left(\frac{(2\,cx-b)\sqrt{\frac{c}{\pi}}}{c}\right)\sin\left(\frac{b^2+4\,ac}{2\,c}\right) - 2\,c^2x^2 + 2\,c\cos\left(cx^2-bx-a\right)\sin\left(cx^2-bx-\right.}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(pi*b*sqrt(c/pi)*cos(1/2*(b^2 + 4*a*c)/c)*fresnel_cos((2*c*x - b)*sqrt
(c/pi)/c) + pi*b*sqrt(c/pi)*fresnel_sin((2*c*x - b)*sqrt(c/pi)/c)*sin(1/2*(
b^2 + 4*a*c)/c) - 2*c^2*x^2 + 2*c*cos(c*x^2 - b*x - a)*sin(c*x^2 - b*x - a)
)/c^2
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int x\sin^2\left(a + bx - cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x**2+b*x+a)**2,x)
```

```
[Out] Integral(x*sin(a + b*x - c*x**2)**2, x)
```

Giac [C]   time = 1.40317, size = 232, normalized size = 1.84

$$\frac{1}{4}x^2 + \frac{\dfrac{\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\right)e^{\left(-\frac{ib^2+4i\,ac}{2c}\right)}}{\sqrt{c}\left(-\frac{ic}{|c|}+1\right)} + i\,e^{\left(2i\,cx^2-2i\,bx-2i\,a\right)}}{16\,c} + \frac{\dfrac{\sqrt{\pi}\,b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\right)e^{\left(-\frac{-ib^2-4i\,ac}{2c}\right)}}{\sqrt{c}\left(\frac{ic}{|c|}+1\right)} - i\,e^{\left(-2i\,cx^2+2i\,bx+2i\,a\right)}}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(-c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 + 1/16*(sqrt(pi)*b*erf(-1/2*sqrt(c)*(2*x - b/c)*(-I*c/abs(c) + 1))*
e^(-1/2*(I*b^2 + 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) + I*e^(2*I*c*x^2 -
 2*I*b*x - 2*I*a))/c + 1/16*(sqrt(pi)*b*erf(-1/2*sqrt(c)*(2*x - b/c)*(I*c/a
bs(c) + 1))*e^(-1/2*(-I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(I*c/abs(c) + 1)) - I*e^
(-2*I*c*x^2 + 2*I*b*x + 2*I*a))/c
```

3.22 $\int \sin^2\left(a + bx - cx^2\right) dx$

Optimal. Leaf size=100

$$\frac{\sqrt{\pi}\cos\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{x}{2}$$

```
[Out] x/2 + (Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[Pi]
)])/(4*Sqrt[c]) + (Sqrt[Pi]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*
a + b^2/(2*c)])/(4*Sqrt[c])
```

Rubi [A]   time = 0.0478904, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3449, 3448, 3352, 3351}

$$\frac{\sqrt{\pi}\cos\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b-2cx}{\sqrt{\pi}\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{b-2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x - c*x^2]^2,x]
```

```
[Out] x/2 + (Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(b - 2*c*x)/(Sqrt[c]*Sqrt[Pi]
)])/(4*Sqrt[c]) + (Sqrt[Pi]*FresnelS[(b - 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*
a + b^2/(2*c)])/(4*Sqrt[c])
```

Rule 3449

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_), x_Symbol] :> Int[ExpandTri
gReduce[Sin[a + b*x + c*x^2]^n, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 1]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

<u>Rule 3351</u>

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

<u>Rubi steps</u>

$$
\begin{aligned}
\int \sin^2\left(a + bx - cx^2\right) dx &= \int \left(\frac{1}{2} - \frac{1}{2}\cos\left(2a + 2bx - 2cx^2\right)\right) dx \\
&= \frac{x}{2} - \frac{1}{2}\int \cos\left(2a + 2bx - 2cx^2\right) dx \\
&= \frac{x}{2} - \frac{1}{2}\cos\left(2a + \frac{b^2}{2c}\right)\int \cos\left(\frac{(2b - 4cx)^2}{8c}\right) dx - \frac{1}{2}\sin\left(2a + \frac{b^2}{2c}\right)\int \sin\left(\frac{(2b - 4cx)^2}{8c}\right) dx \\
&= \frac{x}{2} + \frac{\sqrt{\pi}\cos\left(2a + \frac{b^2}{2c}\right)C\left(\frac{b - 2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}S\left(\frac{b - 2cx}{\sqrt{c}\sqrt{\pi}}\right)\sin\left(2a + \frac{b^2}{2c}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A]   time = 0.109744, size = 102, normalized size = 1.02

$$
\frac{-\sqrt{\pi}\cos\left(2a + \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx - b}{\sqrt{\pi}\sqrt{c}}\right) - \sqrt{\pi}\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{2cx - b}{\sqrt{c}\sqrt{\pi}}\right) + 2\sqrt{c}x}{4\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x - c*x^2]^2,x]
```

```
[Out] (2*Sqrt[c]*x - Sqrt[Pi]*Cos[2*a + b^2/(2*c)]*FresnelC[(-b + 2*c*x)/(Sqrt[c]
*Sqrt[Pi])] - Sqrt[Pi]*FresnelS[(-b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a +
b^2/(2*c)])/(4*Sqrt[c])
```

Maple [A]   time = 0.015, size = 76, normalized size = 0.8

$$
\frac{x}{2} - \frac{\sqrt{\pi}}{4}\left(\cos\left(\frac{4ca + b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx - b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{4ca + b^2}{2c}\right)\text{FresnelS}\left(\frac{2cx - b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right)\right)\frac{1}{\sqrt{c}}
$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(-c*x^2+b*x+a)^2,x)

[Out] 1/2*x-1/4*Pi^(1/2)/c^(1/2)*(cos(1/2*(4*a*c+b^2)/c)*FresnelC(1/Pi^(1/2)/c^(1
/2)*(2*c*x-b))+sin(1/2*(4*a*c+b^2)/c)*FresnelS(1/Pi^(1/2)/c^(1/2)*(2*c*x-b)
))

Maxima [C]   time = 2.07347, size = 410, normalized size = 4.1

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)-i\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+i\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(-c*x^2+b*x+a)^2,x, algorithm="maxima")

[Out] -1/32*(sqrt(2)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(1/2*(b^2 + 4*a*c)/c) + (I*cos(1/4*pi + 1/2*arctan2(0,
 c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c))
 - sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(1/2*(b^2 + 4*a*c)/c))*erf((2*I*c*x
 - I*b)/sqrt(2*I*c)) - ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi + 1/
2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi + 1/2*
arctan2(0, c)))*cos(1/2*(b^2 + 4*a*c)/c) - (I*cos(1/4*pi + 1/2*arctan2(0, c
)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) +
 sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(1/2*(b^2 + 4*a*c)/c))*erf((2*I*c*x -
 I*b)/sqrt(-2*I*c)))*sqrt(abs(c)) - 16*x*abs(c))/abs(c)

Fricas [A]   time = 1.536, size = 235, normalized size = 2.35

$$-\frac{\pi\sqrt{\frac{c}{\pi}}\cos\left(\frac{b^2+4\,ac}{2\,c}\right)\mathrm{C}\left(\frac{(2\,cx-b)\sqrt{\frac{c}{\pi}}}{c}\right)+\pi\sqrt{\frac{c}{\pi}}\,\mathrm{S}\left(\frac{(2\,cx-b)\sqrt{\frac{c}{\pi}}}{c}\right)\sin\left(\frac{b^2+4\,ac}{2\,c}\right)-2\,cx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(-c*x^2+b*x+a)^2,x, algorithm="fricas")

```
[Out] -1/4*(pi*sqrt(c/pi)*cos(1/2*(b^2 + 4*a*c)/c)*fresnel_cos((2*c*x - b)*sqrt(c
/pi)/c) + pi*sqrt(c/pi)*fresnel_sin((2*c*x - b)*sqrt(c/pi)/c)*sin(1/2*(b^2
+ 4*a*c)/c) - 2*c*x)/c
```

Sympy [A]   time = 1.14479, size = 88, normalized size = 0.88

$$\frac{x}{2} - \frac{\sqrt{\pi}\sqrt{-\frac{1}{c}}\left(-\sin\left(2a + \frac{b^2}{2c}\right)S\left(\frac{2b-4cx}{2\sqrt{\pi}\sqrt{-c}}\right) + \cos\left(2a + \frac{b^2}{2c}\right)C\left(\frac{2b-4cx}{2\sqrt{\pi}\sqrt{-c}}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x**2+b*x+a)**2,x)
```

```
[Out] x/2 - sqrt(pi)*sqrt(-1/c)*(-sin(2*a + b**2/(2*c))*fresnels((2*b - 4*c*x)/(2
*sqrt(pi)*sqrt(-c))) + cos(2*a + b**2/(2*c))*fresnelc((2*b - 4*c*x)/(2*sqrt
(pi)*sqrt(-c))))/4
```

Giac [C]   time = 1.34575, size = 167, normalized size = 1.67

$$\frac{1}{2}x + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x - \frac{b}{c}\right)\left(-\frac{ic}{|c|} + 1\right)\right)e^{\left(-\frac{ib^2+4iac}{2c}\right)}}{8\sqrt{c}\left(-\frac{ic}{|c|} + 1\right)} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x - \frac{b}{c}\right)\left(\frac{ic}{|c|} + 1\right)\right)e^{\left(-\frac{-ib^2-4iac}{2c}\right)}}{8\sqrt{c}\left(\frac{ic}{|c|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/8*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - b/c)*(-I*c/abs(c) + 1))*e^(-1/
2*(I*b^2 + 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) + 1/8*sqrt(pi)*erf(-1/2*
sqrt(c)*(2*x - b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 - 4*I*a*c)/c)/(sqrt(c
)*(I*c/abs(c) + 1))
```

3.23 $\int \dfrac{\sin^2\left(a+bx-cx^2\right)}{x}\,dx$

Optimal. Leaf size=32

$$\frac{\log(x)}{2} - \frac{1}{2}\text{Unintegrable}\left(\frac{\cos\left(2a + 2bx - 2cx^2\right)}{x}, x\right)$$

[Out] Log[x]/2 - Unintegrable[Cos[2*a + 2*b*x - 2*c*x^2]/x, x]/2

Rubi [A]   time = 0.0282692, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2\left(a + bx - cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[a + b*x - c*x^2]^2/x,x]

[Out] Log[x]/2 - Defer[Int][Cos[2*a + 2*b*x - 2*c*x^2]/x, x]/2

Rubi steps

$$\int \frac{\sin^2\left(a + bx - cx^2\right)}{x}\,dx = \int\left(\frac{1}{2x} - \frac{\cos\left(2a + 2bx - 2cx^2\right)}{2x}\right)dx$$

$$= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos\left(2a + 2bx - 2cx^2\right)}{x}\,dx$$

Mathematica [A]   time = 7.29916, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a + bx - cx^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + b*x - c*x^2]^2/x,x]

```
[Out] Integrate[Sin[a + b*x - c*x^2]^2/x, x]
```

Maple [A]   time = 0.097, size = 0, normalized size = 0.

$$\int \frac{\left(\sin\left(-cx^2 + bx + a\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(-c*x^2+b*x+a)^2/x,x)
```

```
[Out] int(sin(-c*x^2+b*x+a)^2/x,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}\int \frac{\cos\left(2\,cx^2 - 2\,bx - 2\,a\right)}{x}\, dx + \frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*integrate(cos(2*c*x^2 - 2*b*x - 2*a)/x, x) + 1/2*log(x)
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos\left(cx^2 - bx - a\right)^2 - 1}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(cos(c*x^2 - b*x - a)^2 - 1)/x, x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a + bx - cx^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x**2+b*x+a)**2/x,x)
```

```
[Out] Integral(sin(a + b*x - c*x**2)**2/x, x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(-cx^2 + bx + a\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(-c*x^2+b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(-c*x^2 + b*x + a)^2/x, x)
```

3.24 $\int x^2 \sin^2\left(\frac{1}{4} + x + x^2\right) dx$

Optimal. Leaf size=85

$$-\frac{1}{16}\sqrt{\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{1}{16}\sqrt{\pi}S\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{x^3}{6} - \frac{1}{8}x\sin\left(2x^2 + 2x + \frac{1}{2}\right) + \frac{1}{16}\sin\left(2x^2 + 2x + \frac{1}{2}\right)$$

```
[Out] x^3/6 - (Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/16 + (Sqrt[Pi]*FresnelS[(1
+ 2*x)/Sqrt[Pi]])/16 + Sin[1/2 + 2*x + 2*x^2]/16 - (x*Sin[1/2 + 2*x + 2*x^2
])/8
```

Rubi [A]   time = 0.0619234, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.467, Rules used = {3467, 3464, 3445, 3351, 3462, 3446, 3352}

$$-\frac{1}{16}\sqrt{\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{1}{16}\sqrt{\pi}S\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{x^3}{6} - \frac{1}{8}x\sin\left(2x^2 + 2x + \frac{1}{2}\right) + \frac{1}{16}\sin\left(2x^2 + 2x + \frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[1/4 + x + x^2]^2,x]
```

```
[Out] x^3/6 - (Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/16 + (Sqrt[Pi]*FresnelS[(1
+ 2*x)/Sqrt[Pi]])/16 + Sin[1/2 + 2*x + 2*x^2]/16 - (x*Sin[1/2 + 2*x + 2*x^2
])/8
```

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3464

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_))^(m_), x_Sym
bol] :> Simp[(e*(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2])/(2*c), x] + (-Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Sin[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3445

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Sin[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3446

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Cos[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\int x^2 \sin^2\left(\frac{1}{4} + x + x^2\right) dx = \int \left(\frac{x^2}{2} - \frac{1}{2}x^2 \cos\left(\frac{1}{2} + 2x + 2x^2\right)\right) dx
$$

$$
= \frac{x^3}{6} - \frac{1}{2}\int x^2 \cos\left(\frac{1}{2} + 2x + 2x^2\right) dx
$$

$$
= \frac{x^3}{6} - \frac{1}{8}x\sin\left(\frac{1}{2} + 2x + 2x^2\right) + \frac{1}{8}\int \sin\left(\frac{1}{2} + 2x + 2x^2\right) dx + \frac{1}{4}\int x\cos\left(\frac{1}{2} + 2x + 2x^2\right) dx
$$

$$
= \frac{x^3}{6} + \frac{1}{16}\sin\left(\frac{1}{2} + 2x + 2x^2\right) - \frac{1}{8}x\sin\left(\frac{1}{2} + 2x + 2x^2\right) - \frac{1}{8}\int \cos\left(\frac{1}{2} + 2x + 2x^2\right) dx + \frac{1}{8}\int ...
$$

$$
= \frac{x^3}{6} + \frac{1}{16}\sqrt{\pi}S\left(\frac{1+2x}{\sqrt{\pi}}\right) + \frac{1}{16}\sin\left(\frac{1}{2} + 2x + 2x^2\right) - \frac{1}{8}x\sin\left(\frac{1}{2} + 2x + 2x^2\right) - \frac{1}{8}\int \cos\left(\frac{1}{8}(2...
$$

$$
= \frac{x^3}{6} - \frac{1}{16}\sqrt{\pi}C\left(\frac{1+2x}{\sqrt{\pi}}\right) + \frac{1}{16}\sqrt{\pi}S\left(\frac{1+2x}{\sqrt{\pi}}\right) + \frac{1}{16}\sin\left(\frac{1}{2} + 2x + 2x^2\right) - \frac{1}{8}x\sin\left(\frac{1}{2} + 2x + 2...
$$

Mathematica [A]   time = 0.144895, size = 77, normalized size = 0.91

$$\frac{1}{48}\left(-3\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right)+3\sqrt{\pi}S\left(\frac{2x+1}{\sqrt{\pi}}\right)+8x^3-6x\sin\left(\frac{1}{2}(2x+1)^2\right)+3\sin\left(\frac{1}{2}(2x+1)^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[1/4 + x + x^2]^2,x]
```

```
[Out] (8*x^3 - 3*Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]] + 3*Sqrt[Pi]*FresnelS[(1 +
 2*x)/Sqrt[Pi]] + 3*Sin[(1 + 2*x)^2/2] - 6*x*Sin[(1 + 2*x)^2/2])/48
```

Maple [A]   time = 0.017, size = 64, normalized size = 0.8

$$\frac{x^3}{6}+\frac{1}{16}\sin\left(\frac{1}{2}+2x+2x^2\right)-\frac{x}{8}\sin\left(\frac{1}{2}+2x+2x^2\right)-\frac{\sqrt{\pi}}{16}\mathrm{FresnelC}\left(\frac{1+2x}{\sqrt{\pi}}\right)+\frac{\sqrt{\pi}}{16}\mathrm{FresnelS}\left(\frac{1+2x}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(1/4+x+x^2)^2,x)
```

```
[Out] 1/6*x^3+1/16*sin(1/2+2*x+2*x^2)-1/8*x*sin(1/2+2*x+2*x^2)-1/16*FresnelC((1+2
*x)/Pi^(1/2))*Pi^(1/2)+1/16*FresnelS((1+2*x)/Pi^(1/2))*Pi^(1/2)
```

Maxima [C]   time = 2.32955, size = 231, normalized size = 2.72

$$8192\,x^4+4096\,x^3-x\left(3072i\,e^{\left(2i\,x^2+2i\,x+\frac{1}{2}i\right)}-3072i\,e^{\left(-2i\,x^2-2i\,x-\frac{1}{2}i\right)}\right)-\sqrt{8\,x^2+8\,x+2}\left(-(192i-192)\,\sqrt{2}\sqrt{\pi}\left(\mathrm{erf}\left(\sqrt{2i\,x^2+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(1/4+x+x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24576*(8192*x^4 + 4096*x^3 - x*(3072*I*e^(2*I*x^2 + 2*I*x + 1/2*I) - 3072
*I*e^(-2*I*x^2 - 2*I*x - 1/2*I)) - sqrt(8*x^2 + 8*x + 2)*(-(192*I - 192)*sq
rt(2)*sqrt(pi)*(erf(sqrt(2*I*x^2 + 2*I*x + 1/2*I)) - 1) + (192*I + 192)*sqr
t(2)*sqrt(pi)*(erf(sqrt(-2*I*x^2 - 2*I*x - 1/2*I)) - 1) + (384*I + 384)*sqr
t(2)*gamma(3/2, 2*I*x^2 + 2*I*x + 1/2*I) - (384*I - 384)*sqrt(2)*gamma(3/2,
```

```
 -2*I*x^2 - 2*I*x - 1/2*I)) - 1536*I*e^(2*I*x^2 + 2*I*x + 1/2*I) + 1536*I*e
^(-2*I*x^2 - 2*I*x - 1/2*I))/(2*x + 1)
```

Fricas [A] time = 1.65278, size = 215, normalized size = 2.53

$$\frac{1}{6}x^3 - \frac{1}{8}(2x-1)\cos\left(x^2+x+\frac{1}{4}\right)\sin\left(x^2+x+\frac{1}{4}\right) - \frac{1}{16}\sqrt{\pi}\,\mathrm{C}\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{1}{16}\sqrt{\pi}\,\mathrm{S}\left(\frac{2x+1}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(1/4+x+x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/8*(2*x - 1)*cos(x^2 + x + 1/4)*sin(x^2 + x + 1/4) - 1/16*sqrt(p
i)*fresnel_cos((2*x + 1)/sqrt(pi)) + 1/16*sqrt(pi)*fresnel_sin((2*x + 1)/sq
rt(pi))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sin^2\left(x^2+x+\frac{1}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*sin(1/4+x+x**2)**2,x)
```

```
[Out] Integral(x**2*sin(x**2 + x + 1/4)**2, x)
```

Giac [C] time = 1.35223, size = 86, normalized size = 1.01

$$\frac{1}{6}x^3 - \frac{1}{32}(-2ix+i)e^{\left(2ix^2+2ix+\frac{1}{2}i\right)} - \frac{1}{32}(2ix-i)e^{\left(-2ix^2-2ix-\frac{1}{2}i\right)} + \frac{1}{32}i\sqrt{\pi}\,\mathrm{erf}\left((i-1)x+\frac{1}{2}i-\frac{1}{2}\right) - \frac{1}{32}i\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2*sin(1/4+x+x^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/32*(-2*I*x + I)*e^(2*I*x^2 + 2*I*x + 1/2*I) - 1/32*(2*I*x - I)*
e^(-2*I*x^2 - 2*I*x - 1/2*I) + 1/32*I*sqrt(pi)*erf((I - 1)*x + 1/2*I - 1/2)
```

$$- 1/32*I*sqrt(pi)*erf(-(I + 1)*x - 1/2*I - 1/2)$$

3.25 $\quad \int x \sin^2\left(\frac{1}{4} + x + x^2\right) dx$

Optimal. Leaf size=46

$$\frac{1}{8}\sqrt{\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{x^2}{4} - \frac{1}{8}\sin\left(2x^2 + 2x + \frac{1}{2}\right)$$

[Out] x^2/4 + (Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/8 - Sin[1/2 + 2*x + 2*x^2]/8

Rubi [A]   time = 0.0304473, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3467, 3462, 3446, 3352}

$$\frac{1}{8}\sqrt{\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right) + \frac{x^2}{4} - \frac{1}{8}\sin\left(2x^2 + 2x + \frac{1}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[x*Sin[1/4 + x + x^2]^2,x]

[Out] x^2/4 + (Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/8 - Sin[1/2 + 2*x + 2*x^2]/8

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3446

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Cos[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\int x \sin^2\left(\frac{1}{4} + x + x^2\right) dx &= \int \left(\frac{x}{2} - \frac{1}{2} x \cos\left(\frac{1}{2} + 2x + 2x^2\right)\right) dx \\
&= \frac{x^2}{4} - \frac{1}{2} \int x \cos\left(\frac{1}{2} + 2x + 2x^2\right) dx \\
&= \frac{x^2}{4} - \frac{1}{8} \sin\left(\frac{1}{2} + 2x + 2x^2\right) + \frac{1}{4} \int \cos\left(\frac{1}{2} + 2x + 2x^2\right) dx \\
&= \frac{x^2}{4} - \frac{1}{8} \sin\left(\frac{1}{2} + 2x + 2x^2\right) + \frac{1}{4} \int \cos\left(\frac{1}{8}(2 + 4x)^2\right) dx \\
&= \frac{x^2}{4} + \frac{1}{8} \sqrt{\pi} C\left(\frac{1 + 2x}{\sqrt{\pi}}\right) - \frac{1}{8} \sin\left(\frac{1}{2} + 2x + 2x^2\right)
\end{aligned}
$$

Mathematica [A]   time = 0.0607905, size = 42, normalized size = 0.91

$$
\frac{1}{8}\left(\sqrt{\pi}\text{FresnelC}\left(\frac{2x + 1}{\sqrt{\pi}}\right) + 2x^2 - \sin\left(\frac{1}{2}(2x + 1)^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[1/4 + x + x^2]^2,x]
```

```
[Out] (2*x^2 + Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]] - Sin[(1 + 2*x)^2/2])/8
```

Maple [A]   time = 0.014, size = 35, normalized size = 0.8

$$
\frac{x^2}{4} - \frac{1}{8} \sin\left(\frac{1}{2} + 2x + 2x^2\right) + \frac{\sqrt{\pi}}{8}\text{FresnelC}\left(\frac{1 + 2x}{\sqrt{\pi}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(1/4+x+x^2)^2,x)
```

```
[Out] 1/4*x^2-1/8*sin(1/2+2*x+2*x^2)+1/8*FresnelC((1+2*x)/Pi^(1/2))*Pi^(1/2)
```

Maxima [C]   time = 2.04188, size = 182, normalized size = 3.96

$$\frac{65536\,x^3 + 32768\,x^2 + x\left(16384i\,e^{\left(2i\,x^2+2i\,x+\frac{1}{2}i\right)} - 16384i\,e^{\left(-2i\,x^2-2i\,x-\frac{1}{2}i\right)}\right) + \sqrt{8\,x^2 + 8\,x + 2}\left(-(2048i - 2048)\,\sqrt{2}\sqrt{\pi}\left(\text{erf}\right.\right.}{1310}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/131072*(65536*x^3 + 32768*x^2 + x*(16384*I*e^(2*I*x^2 + 2*I*x + 1/2*I) -
16384*I*e^(-2*I*x^2 - 2*I*x - 1/2*I)) + sqrt(8*x^2 + 8*x + 2)*(-(2048*I - 2
048)*sqrt(2)*sqrt(pi)*(erf(sqrt(2*I*x^2 + 2*I*x + 1/2*I)) - 1) + (2048*I +
2048)*sqrt(2)*sqrt(pi)*(erf(sqrt(-2*I*x^2 - 2*I*x - 1/2*I)) - 1)) + 8192*I*
e^(2*I*x^2 + 2*I*x + 1/2*I) - 8192*I*e^(-2*I*x^2 - 2*I*x - 1/2*I))/(2*x + 1
)
```

Fricas [A]   time = 1.65995, size = 135, normalized size = 2.93

$$\frac{1}{4}\,x^2 - \frac{1}{4}\,\cos\left(x^2 + x + \frac{1}{4}\right)\sin\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{8}\,\sqrt{\pi}\,\text{C}\left(\frac{2\,x+1}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^2 - 1/4*cos(x^2 + x + 1/4)*sin(x^2 + x + 1/4) + 1/8*sqrt(pi)*fresnel_
cos((2*x + 1)/sqrt(pi))
```

Sympy [B]   time = 1.6479, size = 121, normalized size = 2.63

$$\frac{x^2}{4} - \frac{\sqrt{\pi}xC\left(\frac{2x}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi}}\right)}{4} + \frac{\sqrt{\pi}xC\left(\frac{2x}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{16\Gamma\left(\frac{5}{4}\right)} - \frac{\sin\left(2\left(x + \frac{1}{2}\right)^2\right)\Gamma\left(\frac{1}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{\pi}C\left(\frac{2x}{\sqrt{\pi}} + \frac{1}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{32\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x**2)**2,x)
```

```
[Out] x**2/4 - sqrt(pi)*x*fresnelc(2*x/sqrt(pi) + 1/sqrt(pi))/4 + sqrt(pi)*x*fres
nelc(2*x/sqrt(pi) + 1/sqrt(pi))*gamma(1/4)/(16*gamma(5/4)) - sin(2*(x + 1/2
)**2)*gamma(1/4)/(32*gamma(5/4)) + sqrt(pi)*fresnelc(2*x/sqrt(pi) + 1/sqrt(
pi))*gamma(1/4)/(32*gamma(5/4))
```

Giac [C]   time = 1.28864, size = 73, normalized size = 1.59

$$\frac{1}{4}x^2 - \left(\frac{1}{32}i + \frac{1}{32}\right)\sqrt{\pi}\operatorname{erf}\left((i-1)\,x + \frac{1}{2}i - \frac{1}{2}\right) + \left(\frac{1}{32}i - \frac{1}{32}\right)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\,x - \frac{1}{2}i - \frac{1}{2}\right) + \frac{1}{16}i\,e^{\left(2i\,x^2+2i\,x+\frac{1}{2}i\right)} - \frac{1}{16}i\,e^{\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(1/4+x+x^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - (1/32*I + 1/32)*sqrt(pi)*erf((I - 1)*x + 1/2*I - 1/2) + (1/32*I -
 1/32)*sqrt(pi)*erf(-(I + 1)*x - 1/2*I - 1/2) + 1/16*I*e^(2*I*x^2 + 2*I*x +
 1/2*I) - 1/16*I*e^(-2*I*x^2 - 2*I*x - 1/2*I)
```

3.26 $\int \sin^2\left(\frac{1}{4} + x + x^2\right) dx$

Optimal. Leaf size=27

$$\frac{x}{2} - \frac{1}{4}\sqrt{\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right)$$

[Out] x/2 - (Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/4

Rubi [A]   time = 0.0146485, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3449, 3446, 3352}

$$\frac{x}{2} - \frac{1}{4}\sqrt{\pi}\text{FresnelC}\left(\frac{2x+1}{\sqrt{\pi}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sin[1/4 + x + x^2]^2,x]

[Out] x/2 - (Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/4

Rule 3449

Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_), x_Symbol] :> Int[ExpandTri
gReduce[Sin[a + b*x + c*x^2]^n, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 1]

Rule 3446

Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Int[Cos[(b + 2*c*x
)^2/(4*c)], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]

Rule 3352

Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]

Rubi steps

$$\int \sin^2\left(\frac{1}{4} + x + x^2\right) dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos\left(\frac{1}{2} + 2x + 2x^2\right)\right) dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos\left(\frac{1}{2} + 2x + 2x^2\right) dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos\left(\frac{1}{8}(2 + 4x)^2\right) dx$$

$$= \frac{x}{2} - \frac{1}{4}\sqrt{\pi}C\left(\frac{1 + 2x}{\sqrt{\pi}}\right)$$

Mathematica [A]   time = 0.0125249, size = 27, normalized size = 1.

$$\frac{1}{4}\left(2x - \sqrt{\pi}\mathrm{FresnelC}\left(\frac{2x + 1}{\sqrt{\pi}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[1/4 + x + x^2]^2,x]
```

```
[Out] (2*x - Sqrt[Pi]*FresnelC[(1 + 2*x)/Sqrt[Pi]])/4
```

Maple [A]   time = 0.013, size = 20, normalized size = 0.7

$$\frac{x}{2} - \frac{\sqrt{\pi}}{4}\mathrm{FresnelC}\left(\frac{1 + 2x}{\sqrt{\pi}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4+x+x^2)^2,x)
```

```
[Out] 1/2*x-1/4*FresnelC((1+2*x)/Pi^(1/2))*Pi^(1/2)
```

Maxima [C]   time = 1.73747, size = 46, normalized size = 1.7

$$\frac{1}{16}\sqrt{\pi}\left((i - 1)\,\mathrm{erf}\left(\frac{2ix + i}{\sqrt{2i}}\right) + (i + 1)\,\mathrm{erf}\left(\frac{2ix + i}{\sqrt{-2i}}\right)\right) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*((I - 1)*erf((2*I*x + I)/sqrt(2*I)) + (I + 1)*erf((2*I*x + I)
/sqrt(-2*I))) + 1/2*x
```

Fricas [A]   time = 1.68478, size = 74, normalized size = 2.74

$$-\frac{1}{4}\sqrt{\pi}\,\mathrm{C}\left(\frac{2\,x+1}{\sqrt{\pi}}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(pi)*fresnel_cos((2*x + 1)/sqrt(pi)) + 1/2*x
```

Sympy [A]   time = 0.887441, size = 22, normalized size = 0.81

$$\frac{x}{2}-\frac{\sqrt{\pi}C\left(\frac{4x+2}{2\sqrt{\pi}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x**2)**2,x)
```

```
[Out] x/2 - sqrt(pi)*fresnelc((4*x + 2)/(2*sqrt(pi)))/4
```

Giac [C]   time = 1.28109, size = 35, normalized size = 1.3

$$\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\,x+\frac{1}{2}i-\frac{1}{2}\right)-\left(\frac{1}{16}i-\frac{1}{16}\right)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\,x-\frac{1}{2}i-\frac{1}{2}\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2,x, algorithm="giac")
```

```
[Out] (1/16*I + 1/16)*sqrt(pi)*erf((I - 1)*x + 1/2*I - 1/2) - (1/16*I - 1/16)*sqr
t(pi)*erf(-(I + 1)*x - 1/2*I - 1/2) + 1/2*x
```

3.27 $\displaystyle\int \frac{\sin^2\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$

Optimal. Leaf size=30

$$\frac{\log(x)}{2} - \frac{1}{2}\text{Unintegrable}\left(\frac{\cos\left(2x^2+2x+\frac{1}{2}\right)}{x}, x\right)$$

[Out] Log[x]/2 - Unintegrable[Cos[1/2 + 2*x + 2*x^2]/x, x]/2

Rubi [A]   time = 0.0275321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[1/4 + x + x^2]^2/x,x]

[Out] Log[x]/2 - Defer[Int][Cos[1/2 + 2*x + 2*x^2]/x, x]/2

Rubi steps

$$\int \frac{\sin^2\left(\frac{1}{4}+x+x^2\right)}{x}\,dx = \int\left(\frac{1}{2x} - \frac{\cos\left(\frac{1}{2}+2x+2x^2\right)}{2x}\right)dx$$

$$= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos\left(\frac{1}{2}+2x+2x^2\right)}{x}\,dx$$

Mathematica [A]   time = 10.9925, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(\frac{1}{4}+x+x^2\right)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[1/4 + x + x^2]^2/x,x]
```

```
[Out] Integrate[Sin[1/4 + x + x^2]^2/x, x]
```

Maple [A]   time = 0.106, size = 0, normalized size = 0.

$$\int \frac{1}{x} \left(\sin\left(\frac{1}{4} + x + x^2 \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4+x+x^2)^2/x,x)
```

```
[Out] int(sin(1/4+x+x^2)^2/x,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \int \frac{\cos\left(2x^2 + 2x + \frac{1}{2}\right)}{x} dx + \frac{1}{2} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2/x,x, algorithm="maxima")
```

```
[Out] -1/2*integrate(cos(2*x^2 + 2*x + 1/2)/x, x) + 1/2*log(x)
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos\left(x^2 + x + \frac{1}{4}\right)^2 - 1}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2/x,x, algorithm="fricas")
```

```
[Out] integral(-(cos(x^2 + x + 1/4)^2 - 1)/x, x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(x^2 + x + \frac{1}{4}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x**2)**2/x,x)
```

```
[Out] Integral(sin(x**2 + x + 1/4)**2/x, x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2/x,x, algorithm="giac")
```

```
[Out] integrate(sin(x^2 + x + 1/4)^2/x, x)
```

3.28 $\int \dfrac{\sin^2\left(\frac{1}{4}+x+x^2\right)}{x^2}\,dx$

Optimal. Leaf size=65

$$\text{Unintegrable}\left(\frac{\sin\left(2x^2+2x+\frac{1}{2}\right)}{x},x\right)+\sqrt{\pi}S\left(\frac{2x+1}{\sqrt{\pi}}\right)+\frac{\cos\left(2x^2+2x+\frac{1}{2}\right)}{2x}-\frac{1}{2x}$$

```
[Out] -1/(2*x) + Cos[1/2 + 2*x + 2*x^2]/(2*x) + Sqrt[Pi]*FresnelS[(1 + 2*x)/Sqrt[
Pi]] + Unintegrable[Sin[1/2 + 2*x + 2*x^2]/x, x]
```

Rubi [A]   time = 0.0494996, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sin^2\left(\frac{1}{4}+x+x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[1/4 + x + x^2]^2/x^2,x]
```

```
[Out] -1/(2*x) + Cos[1/2 + 2*x + 2*x^2]/(2*x) + Sqrt[Pi]*FresnelS[(1 + 2*x)/Sqrt[
Pi]] + Defer[Int][Sin[1/2 + 2*x + 2*x^2]/x, x]
```

Rubi steps

$$\int \frac{\sin^2\left(\frac{1}{4} + x + x^2\right)}{x^2}\,dx = \int \left(\frac{1}{2x^2} - \frac{\cos\left(\frac{1}{2} + 2x + 2x^2\right)}{2x^2}\right)dx$$

$$= -\frac{1}{2x} - \frac{1}{2}\int \frac{\cos\left(\frac{1}{2} + 2x + 2x^2\right)}{x^2}\,dx$$

$$= -\frac{1}{2x} + \frac{\cos\left(\frac{1}{2} + 2x + 2x^2\right)}{2x} + 2\int \sin\left(\frac{1}{2} + 2x + 2x^2\right)dx + \int \frac{\sin\left(\frac{1}{2} + 2x + 2x^2\right)}{x}\,dx$$

$$= -\frac{1}{2x} + \frac{\cos\left(\frac{1}{2} + 2x + 2x^2\right)}{2x} + 2\int \sin\left(\frac{1}{8}(2 + 4x)^2\right)dx + \int \frac{\sin\left(\frac{1}{2} + 2x + 2x^2\right)}{x}\,dx$$

$$= -\frac{1}{2x} + \frac{\cos\left(\frac{1}{2} + 2x + 2x^2\right)}{2x} + \sqrt{\pi}S\left(\frac{1 + 2x}{\sqrt{\pi}}\right) + \int \frac{\sin\left(\frac{1}{2} + 2x + 2x^2\right)}{x}\,dx$$

Mathematica [A]   time = 11.0357, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(\frac{1}{4} + x + x^2\right)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[1/4 + x + x^2]^2/x^2,x]
```

```
[Out] Integrate[Sin[1/4 + x + x^2]^2/x^2, x]
```

Maple [A]   time = 0.162, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(\sin\left(\frac{1}{4} + x + x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4+x+x^2)^2/x^2,x)
```

```
[Out] int(sin(1/4+x+x^2)^2/x^2,x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$-\frac{x \int \frac{\cos\left(2x^2+2x+\frac{1}{2}\right)}{x^2}\,dx + 1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/4+x+x^2)^2/x^2,x, algorithm="maxima")

[Out] -1/2*(x*integrate(cos(2*x^2 + 2*x + 1/2)/x^2, x) + 1)/x

---

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos\left(x^2+x+\frac{1}{4}\right)^2-1}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/4+x+x^2)^2/x^2,x, algorithm="fricas")

[Out] integral(-(cos(x^2 + x + 1/4)^2 - 1)/x^2, x)

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(x^2+x+\frac{1}{4}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(1/4+x+x**2)**2/x**2,x)

[Out] Integral(sin(x**2 + x + 1/4)**2/x**2, x)

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(x^2 + x + \frac{1}{4}\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4+x+x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sin(x^2 + x + 1/4)^2/x^2, x)
```

3.29 $\int (d + ex)^2 \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=285

$$\frac{\sqrt{\frac{\pi}{2}}\sin\left(a - \frac{b^2}{4c}\right)(2cd - be)^2\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)(2cd - be)^2 S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}}e^2\cos\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(}{2c^{3/2}}$$

```
[Out] -(e*(2*c*d - b*e)*Cos[a + b*x + c*x^2])/(4*c^2) - (e*(d + e*x)*Cos[a + b*x
+ c*x^2])/(2*c) + (e^2*Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(
Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) + ((2*c*d - b*e)^2*Sqrt[Pi/2]*Cos[a - b^2
/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(4*c^(5/2)) + ((2*c*d -
 b*e)^2*Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(
4*c)])/(4*c^(5/2)) - (e^2*Sqrt[Pi/2]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*P
i])]*Sin[a - b^2/(4*c)])/(2*c^(3/2))
```

Rubi [A]   time = 0.270835, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3463, 3448, 3352, 3351, 3461, 3447}

$$\frac{\sqrt{\frac{\pi}{2}}\sin\left(a - \frac{b^2}{4c}\right)(2cd - be)^2\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)(2cd - be)^2 S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{4c^{5/2}} + \frac{\sqrt{\frac{\pi}{2}}e^2\cos\left(a - \frac{b^2}{4c}\right)\text{FresnelC}\left(}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] -(e*(2*c*d - b*e)*Cos[a + b*x + c*x^2])/(4*c^2) - (e*(d + e*x)*Cos[a + b*x
+ c*x^2])/(2*c) + (e^2*Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelC[(b + 2*c*x)/(
Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) + ((2*c*d - b*e)^2*Sqrt[Pi/2]*Cos[a - b^2
/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(4*c^(5/2)) + ((2*c*d -
 b*e)^2*Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(
4*c)])/(4*c^(5/2)) - (e^2*Sqrt[Pi/2]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*P
i])]*Sin[a - b^2/(4*c)])/(2*c^(3/2))
```

Rule 3463

```
Int[((d_.) + (e_.)*(x_))^(m_)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sym
bol] :> -Simp[(e*(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2])/(2*c), x] + (Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Cos[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2], x], x]
```

```
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rubi steps

$$\int (d+ex)^2 \sin\left(a+bx+cx^2\right) dx = -\frac{e(d+ex)\cos\left(a+bx+cx^2\right)}{2c} + \frac{e^2 \int \cos\left(a+bx+cx^2\right) dx}{2c} - \frac{(-2cd+be)\int(d+ex)\,s}{2c}$$

$$= -\frac{e(2cd-be)\cos\left(a+bx+cx^2\right)}{4c^2} - \frac{e(d+ex)\cos\left(a+bx+cx^2\right)}{2c} + \frac{(2cd-be)^2 \int \sin\left(a\right.}{4c^2}$$

$$= -\frac{e(2cd-be)\cos\left(a+bx+cx^2\right)}{4c^2} - \frac{e(d+ex)\cos\left(a+bx+cx^2\right)}{2c} + \frac{e^2\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{b^2}{4c}\right)}{2c^{3/2}}$$

$$= -\frac{e(2cd-be)\cos\left(a+bx+cx^2\right)}{4c^2} - \frac{e(d+ex)\cos\left(a+bx+cx^2\right)}{2c} + \frac{e^2\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{b^2}{4c}\right)}{2c^{3/2}}$$

Mathematica [A]   time = 1.30037, size = 186, normalized size = 0.65

$$\frac{\sqrt{2\pi}\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)\left(\sin\left(a-\frac{b^2}{4c}\right)(be-2cd)^2 + 2ce^2\cos\left(a-\frac{b^2}{4c}\right)\right) + \sqrt{2\pi}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)\left(\cos\left(a-\frac{b^2}{4c}\right)(be-2cd)^2 - 2ce^2\sin\right.}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sin[a + b*x + c*x^2],x]
```

```
[Out] (2*Sqrt[c]*e*(b*e - 2*c*(2*d + e*x))*Cos[a + x*(b + c*x)] + Sqrt[2*Pi]*Fres
nelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*((-2*c*d + b*e)^2*Cos[a - b^2/(4*c)]
 - 2*c*e^2*Sin[a - b^2/(4*c)]) + Sqrt[2*Pi]*FresnelC[(b + 2*c*x)/(Sqrt[c]*S
qrt[2*Pi])]*(2*c*e^2*Cos[a - b^2/(4*c)] + (-2*c*d + b*e)^2*Sin[a - b^2/(4*c
)]))/(8*c^(5/2))
```

Maple [A]   time = 0.01, size = 399, normalized size = 1.4

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*sin(c*x^2+b*x+a),x)
```

```
[Out] -1/2*e^2/c*x*cos(c*x^2+b*x+a)-1/2*e^2*b/c*(-1/2*cos(c*x^2+b*x+a)/c-1/4*b/c^
(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1
/2)*(c*x+1/2*b))-sin((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*
x+1/2*b))))+1/4*e^2/c^(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelC
```

```
(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b))+sin((1/4*b^2-c*a)/c)*FresnelS(2^(1/2
)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b)))-d*e/c*cos(c*x^2+b*x+a)-1/2*d*e*b/c^(3/2)*2
^(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*
x+1/2*b))-sin((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b
)))+1/2*2^(1/2)*Pi^(1/2)/c^(1/2)*d^2*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)
/Pi^(1/2)/c^(1/2)*(c*x+1/2*b))-sin((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/
2)/c^(1/2)*(c*x+1/2*b)))
```

Maxima [C]   time = 3.95157, size = 5825, normalized size = 20.44

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*pi + 1/2*arctan2
(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) - (cos(1/4*pi + 1/2*arctan2(0, c)) + cos
(-1/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-
1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x +
I*b)/sqrt(I*c)) + ((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/
2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arct
an2(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) +
cos(-1/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*si
n(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x
 + I*b)/sqrt(-I*c)))*d^2/sqrt(abs(c)) - 1/4*(((I*sqrt(pi)*(erf(1/2*sqrt((4*
I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) - I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(-1/4*(b^2 - 4*a*c)/c) + (sqrt(
pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(er
f(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/4*(b^2 -
 4*a*c)/c) + ((2*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)
/c)) - 1) - 2*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c
)) - 1))*b*c*cos(-1/4*(b^2 - 4*a*c)/c) + 2*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 +
 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/4*(b^2 - 4*a*c)/c))*x)*cos(1/2*arc
tan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + ((sqrt(pi)*(erf(1/2*sqrt((4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 +
 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(-1/4*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*
(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(
1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/4*(b^2 - 4
*a*c)/c) + (2*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
```

```
   - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))
*b*c*cos(-1/4*(b^2 - 4*a*c)/c) + (-2*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 +
4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/4*(b^2 - 4*a*c)/c))*x)*sin(1/2*arct
an2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + (2*c*(e^(1/4*(4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c) + e^(-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*cos(-1/4*(b
^2 - 4*a*c)/c) + c*(2*I*e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 2*I*e
^(-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*sin(-1/4*(b^2 - 4*a*c)/c))*sqr
t((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c)))*d*e/(c^2*sqrt((4*c^2*x^2 + 4*b*c*x +
 b^2)/abs(c))) + 1/16*((4*b*c*(e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)
+ e^(-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*
c)/c) + b*c*(4*I*e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 4*I*e^(-1/4*
(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*((4
*c^2*x^2 + 4*b*c*x + b^2)/abs(c))^(3/2) + (b^3*(-4*I*gamma(3/2, 1/4*(4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c) + 4*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*
x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - 4*b^3*(gamma(3/2, 1/4*(4*
I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*
x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c) + (c^3*(-32*I*gamma(3/2, 1/
4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + 32*I*gamma(3/2, -1/4*(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - 32*c^3*(gamma(3
/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2
 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*x^3 + (b*c^2*(-
48*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + 48*I*gamma(3/2,
-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c)
 - 48*b*c^2*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/
2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)
/c))*x^2 + (b^2*c*(-24*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c
) + 24*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-
1/4*(b^2 - 4*a*c)/c) - 24*b^2*c*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x +
I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*si
n(-1/4*(b^2 - 4*a*c)/c))*x)*cos(3/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c,
0)) + ((I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1)
 - I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^
5*cos(-1/4*(b^2 - 4*a*c)/c) + (sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x +
I*b^2)/c)) - 1))*b^5*sin(-1/4*(b^2 - 4*a*c)/c) + ((8*I*sqrt(pi)*(erf(1/2*sq
rt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) - 8*I*sqrt(pi)*(erf(1/2*sqrt(
-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/4*(b^2 - 4*a*c)
/c) + 8*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1)
+ sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c
^3*sin(-1/4*(b^2 - 4*a*c)/c))*x^3 + ((12*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*
x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) - 12*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x
^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/4*(b^2 - 4*a*c)/c) + 12*(s
qrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)
*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(-1/
```

```
4*(b^2 - 4*a*c)/c))*x^2 + ((6*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b
*c*x + I*b^2)/c)) - 1) - 6*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c
*x + I*b^2)/c)) - 1))*b^4*c*cos(-1/4*(b^2 - 4*a*c)/c) + 6*(sqrt(pi)*(erf(1/
2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(
-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/4*(b^2 - 4*a*c)/c
))*x)*cos(1/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) - (4*b^3*(gamma(3/
2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - b^3*(4*I*gamma(
3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 4*I*gamma(3/2, -1/4*(4*I*c^
2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c) + (32*c^3*(
gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/4*(4*I*
c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*a*c)/c) - c^3*(32
*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 32*I*gamma(3/2, -1
/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*
x^3 + (48*b*c^2*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamm
a(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/4*(b^2 - 4*
a*c)/c) - b*c^2*(48*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) -
 48*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/4
*(b^2 - 4*a*c)/c))*x^2 + (24*b^2*c*(gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*b*c*x
 + I*b^2)/c) + gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)
*cos(-1/4*(b^2 - 4*a*c)/c) - b^2*c*(24*I*gamma(3/2, 1/4*(4*I*c^2*x^2 + 4*I*
b*c*x + I*b^2)/c) - 24*I*gamma(3/2, -1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/
c))*abs(c)*sin(-1/4*(b^2 - 4*a*c)/c))*x)*sin(3/2*arctan2((4*c^2*x^2 + 4*b*c
*x + b^2)/c, 0)) + ((sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
 - 1))*b^5*cos(-1/4*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*
x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(-1/4*(b^2 - 4*a*c)/c) + (8*(sqrt(pi)
*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1
/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/4*(b^2
- 4*a*c)/c) + (-8*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2
)/c)) - 1) + 8*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/
c)) - 1))*b^2*c^3*sin(-1/4*(b^2 - 4*a*c)/c))*x^3 + (12*(sqrt(pi)*(erf(1/2*s
qrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/4*(b^2 - 4*a*c)/c)
 + (-12*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1)
 + 12*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))
*b^3*c^2*sin(-1/4*(b^2 - 4*a*c)/c))*x^2 + (6*(sqrt(pi)*(erf(1/2*sqrt((4*I*c
^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2
 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(-1/4*(b^2 - 4*a*c)/c) + (-6*I*sqr
t(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 6*I*sqrt(p
i)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/
4*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)))*
e^2/(c^3*((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c))^(3/2)*abs(c))
```

Fricas [A]   time = 1.79653, size = 567, normalized size = 1.99

$$\sqrt{2}\Big(2\,\pi ce^2\cos\Big(-\tfrac{b^2-4\,ac}{4\,c}\Big)+\pi\Big(4\,c^2d^2-4\,bcde+b^2e^2\Big)\sin\Big(-\tfrac{b^2-4\,ac}{4\,c}\Big)\Big)\sqrt{\tfrac{c}{\pi}}\,\mathrm{C}\Bigg(\tfrac{\sqrt{2}(2\,cx+b)\sqrt{\tfrac{c}{\pi}}}{2\,c}\Bigg)-\sqrt{2}\Big(2\,\pi ce^2\sin\Big(-\tfrac{b^2-4\,ac}{4\,c}\Big)-\pi\Big($$

$$\frac{}{8\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(2*pi*c*e^2*cos(-1/4*(b^2 - 4*a*c)/c) + pi*(4*c^2*d^2 - 4*b*c*
d*e + b^2*e^2)*sin(-1/4*(b^2 - 4*a*c)/c))*sqrt(c/pi)*fresnel_cos(1/2*sqrt(2
)*(2*c*x + b)*sqrt(c/pi)/c) - sqrt(2)*(2*pi*c*e^2*sin(-1/4*(b^2 - 4*a*c)/c)
 - pi*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*cos(-1/4*(b^2 - 4*a*c)/c))*sqrt(c/p
i)*fresnel_sin(1/2*sqrt(2)*(2*c*x + b)*sqrt(c/pi)/c) - 2*(2*c^2*e^2*x + 4*c
^2*d*e - b*c*e^2)*cos(c*x^2 + b*x + a))/c^3
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int (d+ex)^2\sin\left(a+bx+cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral((d + e*x)**2*sin(a + b*x + c*x**2), x)
```

Giac [C]   time = 1.2609, size = 768, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*s
qrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c)))
```

```
 - 1/4*I*sqrt(2)*sqrt(pi)*d^2*erf(-1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*
sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
 - 1/4*(I*sqrt(2)*sqrt(pi)*b*d*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) +
1)*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c - 4*c)/c)/((-I*c/abs(c) + 1)*sqrt
(abs(c))) + 2*d*e^(I*c*x^2 + I*b*x + I*a + 1))/c - 1/4*(-I*sqrt(2)*sqrt(pi)
*b*d*erf(-1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-
I*b^2 + 4*I*a*c - 4*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) + 2*d*e^(-I*c*x^2
 - I*b*x - I*a + 1))/c - 1/16*(-I*sqrt(2)*sqrt(pi)*(b^2 + 2*I*c)*erf(-1/4*s
qrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c
 - 8*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) - 2*I*(c*(2*I*x + I*b/c) - 2*I*
b)*e^(I*c*x^2 + I*b*x + I*a + 2))/c^2 - 1/16*(I*sqrt(2)*sqrt(pi)*(b^2 - 2*I
*c)*erf(-1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I
*b^2 + 4*I*a*c - 8*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) - 2*I*(c*(2*I*x +
I*b/c) - 2*I*b)*e^(-I*c*x^2 - I*b*x - I*a + 2))/c^2
```

3.30 $\int (d + ex) \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=140

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(a - \frac{b^2}{4c}\right)(2cd - be)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right)(2cd - be)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{e \cos\left(a + bx + cx^2\right)}{2c}$$

```
[Out] -(e*Cos[a + b*x + c*x^2])/(2*c) + ((2*c*d - b*e)*Sqrt[Pi/2]*Cos[a - b^2/(4*
c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) + ((2*c*d - b*e
)*Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])
/(2*c^(3/2))
```

Rubi [A]   time = 0.0558215, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.235, Rules used = {3461, 3447, 3351, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(a - \frac{b^2}{4c}\right)(2cd - be)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{2c^{3/2}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right)(2cd - be)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} - \frac{e \cos\left(a + bx + cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sin[a + b*x + c*x^2],x]
```

```
[Out] -(e*Cos[a + b*x + c*x^2])/(2*c) + ((2*c*d - b*e)*Sqrt[Pi/2]*Cos[a - b^2/(4*
c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/(2*c^(3/2)) + ((2*c*d - b*e
)*Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])
/(2*c^(3/2))
```

Rule 3461

```
Int[((d_.) + (e_.)*(x_))*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol]
:> -Simp[(e*Cos[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int
[Sin[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b
*e, 0]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
```

```
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (d + ex)\sin\left(a + bx + cx^2\right)\,dx = -\frac{e\cos\left(a + bx + cx^2\right)}{2c} + \frac{(2cd - be)\int \sin\left(a + bx + cx^2\right)\,dx}{2c}$$

$$= -\frac{e\cos\left(a + bx + cx^2\right)}{2c} + \frac{\left((2cd - be)\cos\left(a - \frac{b^2}{4c}\right)\right)\int \sin\left(\frac{(b+2cx)^2}{4c}\right)\,dx}{2c} + \frac{\left((2cd - be)\text{s}\right.}{}$$

$$= -\frac{e\cos\left(a + bx + cx^2\right)}{2c} + \frac{(2cd - be)\sqrt{\frac{\pi}{2}}\cos\left(a - \frac{b^2}{4c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{2c^{3/2}} + \frac{(2cd - be)\sqrt{\frac{\pi}{2}}C\left(\frac{b+}{\sqrt{c}}\right)}{2c^{3,}}$$

Mathematica [A]   time = 0.668156, size = 128, normalized size = 0.91

$$\frac{\sqrt{2\pi}\sin\left(a - \frac{b^2}{4c}\right)(2cd - be)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right) + \sqrt{2\pi}\cos\left(a - \frac{b^2}{4c}\right)(2cd - be)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right) - 2\sqrt{c}e\cos(a + x(b + cx))}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sin[a + b*x + c*x^2],x]
```

```
[Out] (-2*Sqrt[c]*e*Cos[a + x*(b + c*x)] + (2*c*d - b*e)*Sqrt[2*Pi]*Cos[a - b^2/(
4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])] + (2*c*d - b*e)*Sqrt[2*Pi]
*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)])/(4*c^(3/2))
```

Maple [A]   time = 0.01, size = 184, normalized size = 1.3

$$-\frac{e\cos\left(cx^2 + bx + a\right)}{2c} - \frac{be\sqrt{2}\sqrt{\pi}}{4}\left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right)\text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right)\text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*sin(c*x^2+b*x+a),x)
```

```
[Out] -1/2*e*cos(c*x^2+b*x+a)/c-1/4*e*b/c^(3/2)*2^(1/2)*Pi^(1/2)*(cos((1/4*b^2-c*
a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b))-sin((1/4*b^2-c*a)/c)*F
resnelC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b)))+1/2*2^(1/2)*Pi^(1/2)/c^(1/2)
*d*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b))-sin
((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/2)*(c*x+1/2*b)))
```

Maxima [C]   time = 2.71412, size = 1702, normalized size = 12.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(pi)*((((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*pi + 1/2*arctan2
(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) - (cos(1/4*pi + 1/2*arctan2(0, c)) + cos
(-1/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-
1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x +
I*b)/sqrt(I*c)) + ((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/
2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arct
an2(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) +
cos(-1/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*si
n(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x
 + I*b)/sqrt(-I*c)))*d/sqrt(abs(c)) - 1/8*(((I*sqrt(pi)*(erf(1/2*sqrt((4*I*
c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) - I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*
x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(-1/4*(b^2 - 4*a*c)/c) + (sqrt(pi
)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(
1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/4*(b^2 - 4
*a*c)/c) + ((2*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c
)) - 1) - 2*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
 - 1))*b*c*cos(-1/4*(b^2 - 4*a*c)/c) + 2*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x
^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/4*(b^2 - 4*a*c)/c))*x)*cos(1/2*arcta
n2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + ((sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x
^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c)) - 1))*b^2*cos(-1/4*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(e
rf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(1/
```

```
2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/4*(b^2 - 4*a
*c)/c) + (2*(sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) -
 1) + sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b
*c*cos(-1/4*(b^2 - 4*a*c)/c) + (-2*I*sqrt(pi)*(erf(1/2*sqrt((4*I*c^2*x^2 +
4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(erf(1/2*sqrt(-(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/4*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan
2((4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + (2*c*(e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*
x + I*b^2)/c) + e^(-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*cos(-1/4*(b^2
 - 4*a*c)/c) + c*(2*I*e^(1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 2*I*e^(
-1/4*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*sin(-1/4*(b^2 - 4*a*c)/c))*sqrt(
(4*c^2*x^2 + 4*b*c*x + b^2)/abs(c)))*e/(c^2*sqrt((4*c^2*x^2 + 4*b*c*x + b^2
)/abs(c)))
```

Fricas [A]   time = 1.74135, size = 359, normalized size = 2.56

$$\frac{\sqrt{2}\pi(2\,cd - be)\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{4\,c}\right)\mathrm{S}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right) + \sqrt{2}\pi(2\,cd - be)\sqrt{\frac{c}{\pi}}\,\mathrm{C}\left(\frac{\sqrt{2}(2\,cx+b)\sqrt{\frac{c}{\pi}}}{2\,c}\right)\sin\left(-\frac{b^2-4\,ac}{4\,c}\right) - 2\,ce\cos\left(cx^2 + \right.}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*pi*(2*c*d - b*e)*sqrt(c/pi)*cos(-1/4*(b^2 - 4*a*c)/c)*fresnel_
sin(1/2*sqrt(2)*(2*c*x + b)*sqrt(c/pi)/c) + sqrt(2)*pi*(2*c*d - b*e)*sqrt(c
/pi)*fresnel_cos(1/2*sqrt(2)*(2*c*x + b)*sqrt(c/pi)/c)*sin(-1/4*(b^2 - 4*a*
c)/c) - 2*c*e*cos(c*x^2 + b*x + a))/c^2
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int (d + ex)\sin\left(a + bx + cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x**2+b*x+a),x)
```

```
[Out] Integral((d + e*x)*sin(a + b*x + c*x**2), x)
```

Giac [C]   time = 1.28206, size = 441, normalized size = 3.15

$$\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}} - \frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}} - \frac{i\sqrt{2}\sqrt{\pi}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*sin(c*x^2+b*x+a),x, algorithm="giac")

[Out] 1/4*I*sqrt(2)*sqrt(pi)*d*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*sqr
t(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) -
1/4*I*sqrt(2)*sqrt(pi)*d*erf(-1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt
(abs(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) - 1
/8*(I*sqrt(2)*sqrt(pi)*b*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*sqr
t(abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c - 4*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c
))) + 2*e^(I*c*x^2 + I*b*x + I*a + 1))/c - 1/8*(-I*sqrt(2)*sqrt(pi)*b*erf(-
1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt(abs(c)))*e^(-1/4*(-I*b^2 + 4*
I*a*c - 4*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c))) + 2*e^(-I*c*x^2 - I*b*x - I
*a + 1))/c

3.31 $\quad \int \sin\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=97

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}}$$

```
[Out] (Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/
Sqrt[c] + (Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^
2/(4*c)])/Sqrt[c]
```

Rubi [A]   time = 0.0221977, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3447, 3351, 3352}

$$\frac{\sqrt{\frac{\pi}{2}} \sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])])/
Sqrt[c] + (Sqrt[Pi/2]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^
2/(4*c)])/Sqrt[c]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int \sin\left(a + bx + cx^2\right) dx = \cos\left(a - \frac{b^2}{4c}\right) \int \sin\left(\frac{(b + 2cx)^2}{4c}\right) dx + \sin\left(a - \frac{b^2}{4c}\right) \int \cos\left(\frac{(b + 2cx)^2}{4c}\right) dx$$

$$= \frac{\sqrt{\frac{\pi}{2}} \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)}{\sqrt{c}} + \frac{\sqrt{\frac{\pi}{2}} C\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right) \sin\left(a - \frac{b^2}{4c}\right)}{\sqrt{c}}$$

Mathematica [A]   time = 0.0951205, size = 84, normalized size = 0.87

$$\frac{\sqrt{\frac{\pi}{2}} \left(\sin\left(a - \frac{b^2}{4c}\right) \text{FresnelC}\left(\frac{b+2cx}{\sqrt{2\pi}\sqrt{c}}\right) + \cos\left(a - \frac{b^2}{4c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{2\pi}}\right)\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi/2]*(Cos[a - b^2/(4*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]
+ FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[2*Pi])]*Sin[a - b^2/(4*c)]))/Sqrt[c]
```

Maple [A]   time = 0.007, size = 82, normalized size = 0.9

$$\frac{\sqrt{2}\sqrt{\pi}}{2} \left(\cos\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right) \text{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right) - \sin\left(\frac{1}{c}\left(\frac{b^2}{4} - ca\right)\right) \text{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\left(cx + \frac{b}{2}\right)\frac{1}{\sqrt{c}}\right)\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*x^2+b*x+a),x)
```

```
[Out] 1/2*2^(1/2)*Pi^(1/2)/c^(1/2)*(cos((1/4*b^2-c*a)/c)*FresnelS(2^(1/2)/Pi^(1/2
)/c^(1/2)*(c*x+1/2*b))-sin((1/4*b^2-c*a)/c)*FresnelC(2^(1/2)/Pi^(1/2)/c^(1/
2)*(c*x+1/2*b)))
```

Maxima [C]   time = 2.04684, size = 393, normalized size = 4.05

$$-\frac{\sqrt{\pi}\left(\left(\left(-i\,\cos\left(\tfrac{1}{4}\pi+\tfrac{1}{2}\arctan(0,c)\right)-i\,\cos\left(-\tfrac{1}{4}\pi+\tfrac{1}{2}\arctan(0,c)\right)-\sin\left(\tfrac{1}{4}\pi+\tfrac{1}{2}\arctan(0,c)\right)+\sin\left(-\tfrac{1}{4}\pi+\tfrac{1}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out]  -1/8*sqrt(pi)*(((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/2*a
rctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)) + sin(-1/4*pi + 1/2*arctan2
(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) - (cos(1/4*pi + 1/2*arctan2(0, c)) + cos
(-1/4*pi + 1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-
1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x +
I*b)/sqrt(I*c)) + ((-I*cos(1/4*pi + 1/2*arctan2(0, c)) - I*cos(-1/4*pi + 1/
2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)) - sin(-1/4*pi + 1/2*arct
an2(0, c)))*cos(-1/4*(b^2 - 4*a*c)/c) + (cos(1/4*pi + 1/2*arctan2(0, c)) +
cos(-1/4*pi + 1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*si
n(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/4*(b^2 - 4*a*c)/c))*erf(1/2*(2*I*c*x
 + I*b)/sqrt(-I*c)))/sqrt(abs(c))
```

Fricas [A]   time = 1.77984, size = 279, normalized size = 2.88

$$\frac{\sqrt{2}\pi\sqrt{\tfrac{c}{\pi}}\cos\left(-\tfrac{b^2-4ac}{4c}\right)S\left(\frac{\sqrt{2}(2cx+b)\sqrt{\tfrac{c}{\pi}}}{2c}\right)+\sqrt{2}\pi\sqrt{\tfrac{c}{\pi}}\,C\left(\frac{\sqrt{2}(2cx+b)\sqrt{\tfrac{c}{\pi}}}{2c}\right)\sin\left(-\tfrac{b^2-4ac}{4c}\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sin(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out]  1/2*(sqrt(2)*pi*sqrt(c/pi)*cos(-1/4*(b^2 - 4*a*c)/c)*fresnel_sin(1/2*sqrt(2
)*(2*c*x + b)*sqrt(c/pi)/c) + sqrt(2)*pi*sqrt(c/pi)*fresnel_cos(1/2*sqrt(2)
*(2*c*x + b)*sqrt(c/pi)/c)*sin(-1/4*(b^2 - 4*a*c)/c))/c
```

Sympy [A]   time = 0.580319, size = 88, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{\pi}\left(\sin\left(a-\tfrac{b^2}{4c}\right)C\left(\frac{\sqrt{2}(b+2cx)}{2\sqrt{\pi}\sqrt{c}}\right)+\cos\left(a-\tfrac{b^2}{4c}\right)S\left(\frac{\sqrt{2}(b+2cx)}{2\sqrt{\pi}\sqrt{c}}\right)\right)\sqrt{\tfrac{1}{c}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x**2+b*x+a),x)
```

```
[Out] sqrt(2)*sqrt(pi)*(sin(a - b**2/(4*c))*fresnelc(sqrt(2)*(b + 2*c*x)/(2*sqrt(
pi)*sqrt(c))) + cos(a - b**2/(4*c))*fresnels(sqrt(2)*(b + 2*c*x)/(2*sqrt(pi
)*sqrt(c))))*sqrt(1/c)/2
```

Giac [C]　　time = 1.21581, size = 182, normalized size = 1.88

$$\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{ib^2-4iac}{4c}\right)}}{4\left(-\frac{ic}{|c|}+1\right)\sqrt{|c|}}-\frac{i\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\left(2x+\frac{b}{c}\right)\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}\right)e^{\left(-\frac{-ib^2+4iac}{4c}\right)}}{4\left(\frac{ic}{|c|}+1\right)\sqrt{|c|}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + b/c)*(-I*c/abs(c) + 1)*sqrt(
abs(c)))*e^(-1/4*(I*b^2 - 4*I*a*c)/c)/((-I*c/abs(c) + 1)*sqrt(abs(c))) - 1/
4*I*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*(2*x + b/c)*(I*c/abs(c) + 1)*sqrt(abs
(c)))*e^(-1/4*(-I*b^2 + 4*I*a*c)/c)/((I*c/abs(c) + 1)*sqrt(abs(c)))
```

3.32 $\int \frac{\sin\left(a+bx+cx^2\right)}{d+ex}\,dx$

Optimal. Leaf size=21

$$\text{Unintegrable}\left(\frac{\sin\left(a+bx+cx^2\right)}{d+ex},x\right)$$

[Out] Unintegrable[Sin[a + b*x + c*x^2]/(d + e*x), x]

Rubi [A]   time = 0.0113595, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin\left(a+bx+cx^2\right)}{d+ex}\,dx$$

Verification is Not applicable to the result.

[In] Int[Sin[a + b*x + c*x^2]/(d + e*x),x]

[Out] Defer[Int][Sin[a + b*x + c*x^2]/(d + e*x), x]

Rubi steps

$$\int \frac{\sin\left(a+bx+cx^2\right)}{d+ex}\,dx = \int \frac{\sin\left(a+bx+cx^2\right)}{d+ex}\,dx$$

Mathematica [A]   time = 7.65171, size = 0, normalized size = 0.

$$\int \frac{\sin\left(a+bx+cx^2\right)}{d+ex}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[Sin[a + b*x + c*x^2]/(d + e*x),x]

[Out] Integrate[Sin[a + b*x + c*x^2]/(d + e*x), x]

Maple [A]   time = 0.115, size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c*x^2+b*x+a)/(e*x+d),x)

[Out] int(sin(c*x^2+b*x+a)/(e*x+d),x)

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")

[Out] integrate(sin(c*x^2 + b*x + a)/(e*x + d), x)

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sin\left(cx^2 + bx + a\right)}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")

[Out] integral(sin(c*x^2 + b*x + a)/(e*x + d), x)

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(a + bx + cx^2\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x**2+b*x+a)/(e*x+d),x)
```

```
[Out] Integral(sin(a + b*x + c*x**2)/(d + e*x), x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)}{ex + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sin(c*x^2 + b*x + a)/(e*x + d), x)
```

3.33 $\int (d + ex)^2 \sin^2\left(a + bx + cx^2\right)\, dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)(2cd - be)^2\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)(2cd - be)^2 S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}e^2\sin\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}}{16c^{3/2}}$$

```
[Out] (d + e*x)^3/(6*e) - ((2*c*d - b*e)^2*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC
[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])])/(16*c^(5/2)) + (e^2*Sqrt[Pi]*Cos[2*a - b^
2/(2*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])])/(16*c^(3/2)) + (e^2*Sqrt
[Pi]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(16*c^(
3/2)) + ((2*c*d - b*e)^2*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*
Sin[2*a - b^2/(2*c)])/(16*c^(5/2)) - (e*(2*c*d - b*e)*Sin[2*a + 2*b*x + 2*c
*x^2])/(16*c^2) - (e*(d + e*x)*Sin[2*a + 2*b*x + 2*c*x^2])/(8*c)
```

Rubi [A]   time = 0.337417, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3467, 3464, 3447, 3351, 3352, 3462, 3448}

$$-\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)(2cd - be)^2\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)(2cd - be)^2 S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} + \frac{\sqrt{\pi}e^2\sin\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (d + e*x)^3/(6*e) - ((2*c*d - b*e)^2*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC
[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])])/(16*c^(5/2)) + (e^2*Sqrt[Pi]*Cos[2*a - b^
2/(2*c)]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])])/(16*c^(3/2)) + (e^2*Sqrt
[Pi]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(16*c^(
3/2)) + ((2*c*d - b*e)^2*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*
Sin[2*a - b^2/(2*c)])/(16*c^(5/2)) - (e*(2*c*d - b*e)*Sin[2*a + 2*b*x + 2*c
*x^2])/(16*c^2) - (e*(d + e*x)*Sin[2*a + 2*b*x + 2*c*x^2])/(8*c)
```

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3464

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_))^(m_), x_Sym
bol] :> Simp[(e*(d + e*x)^(m - 1)*Sin[a + b*x + c*x^2])/(2*c), x] + (-Dist[
(e^2*(m - 1))/(2*c), Int[(d + e*x)^(m - 2)*Sin[a + b*x + c*x^2], x], x] - D
ist[(b*e - 2*c*d)/(2*c), Int[(d + e*x)^(m - 1)*Cos[a + b*x + c*x^2], x], x]
) /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] && GtQ[m, 1]
```

Rule 3447

```
Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] - Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rubi steps

$$\int (d + ex)^2 \sin^2\left(a + bx + cx^2\right) dx = \int \left(\frac{1}{2}(d + ex)^2 - \frac{1}{2}(d + ex)^2 \cos\left(2a + 2bx + 2cx^2\right)\right) dx$$

$$= \frac{(d + ex)^3}{6e} - \frac{1}{2}\int (d + ex)^2 \cos\left(2a + 2bx + 2cx^2\right) dx$$

$$= \frac{(d + ex)^3}{6e} - \frac{e(d + ex)\sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{e^2 \int \sin\left(2a + 2bx + 2cx^2\right) dx}{8c} - \frac{(2cd}{}$$

$$= \frac{(d + ex)^3}{6e} - \frac{e(2cd - be)\sin\left(2a + 2bx + 2cx^2\right)}{16c^2} - \frac{e(d + ex)\sin\left(2a + 2bx + 2cx^2\right)}{8c} - \frac{(2cd -}{}$$

$$= \frac{(d + ex)^3}{6e} + \frac{e^2\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{3/2}} + \frac{e^2\sqrt{\pi}C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)\sin\left(2a - \frac{b^2}{2c}\right)}{16c^{3/2}} - \frac{e(2cd -}{}$$

$$= \frac{(d + ex)^3}{6e} - \frac{(2cd - be)^2\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right) C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{5/2}} + \frac{e^2\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right) S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{16c^{3/2}}$$

Mathematica [A]   time = 1.18987, size = 215, normalized size = 0.74

$$\frac{-3\sqrt{\pi}\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)\left(\cos\left(2a - \frac{b^2}{2c}\right)(be - 2cd)^2 - ce^2\sin\left(2a - \frac{b^2}{2c}\right)\right) + 3\sqrt{\pi}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)\left(\sin\left(2a - \frac{b^2}{2c}\right)(be - 2cd)^2 + ce^2\,c}{48c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (-3*Sqrt[Pi]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*((-2*c*d + b*e)^2*Cos
[2*a - b^2/(2*c)] - c*e^2*Sin[2*a - b^2/(2*c)]) + 3*Sqrt[Pi]*FresnelS[(b +
2*c*x)/(Sqrt[c]*Sqrt[Pi])]*(c*e^2*Cos[2*a - b^2/(2*c)] + (-2*c*d + b*e)^2*S
in[2*a - b^2/(2*c)]) + Sqrt[c]*(8*c^2*x*(3*d^2 + 3*d*e*x + e^2*x^2) - 3*e*(
4*c*d - b*e + 2*c*e*x)*Sin[2*(a + x*(b + c*x))]))/(48*c^(5/2))
```

Maple [A]   time = 0.017, size = 378, normalized size = 1.3

$$-\frac{e^2 x \sin\left(2cx^2 + 2bx + 2a\right)}{8c} + \frac{e^2 b}{4c}\left(\frac{\sin\left(2cx^2 + 2bx + 2a\right)}{4c} - \frac{b\sqrt{\pi}}{4}\left(\cos\left(\frac{-4ca + b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{-}{}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*sin(c*x^2+b*x+a)^2,x)
```

```
[Out] -1/8*e^2/c*x*sin(2*c*x^2+2*b*x+2*a)+1/4*e^2*b/c*(1/4*sin(2*c*x^2+2*b*x+2*a)
/c-1/4*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/
2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*FresnelS((2*c*x+b)/c^(1/2)/Pi^(1/2))))
+1/16*e^2/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a*c+b^2)/c)*FresnelS((2*c*x+b)/c^(1
/2)/Pi^(1/2))-sin(1/2*(-4*a*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2)))
-1/4*d*e/c*sin(2*c*x^2+2*b*x+2*a)+1/4*d*e*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a
*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*Fre
snelS((2*c*x+b)/c^(1/2)/Pi^(1/2)))-1/4*Pi^(1/2)/c^(1/2)*d^2*(cos(1/2*(-4*a*
c+b^2)/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*Fres
nelS((2*c*x+b)/c^(1/2)/Pi^(1/2)))+1/2*x^2*d*e+1/2*d^2*x+1/6*x^3*e^2
```

Maxima [C]   time = 3.79014, size = 6030, normalized size = 20.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*sin(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(2)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) - (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)
) - sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(2*I*c)) - ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) + (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)
) + sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(-2*I*c)))*sqrt(abs(c)) - 16*x*abs(c))*d^2/abs(c) + 1/16*sqrt
(2)*(sqrt(2)*(4*c^2*x^2 + c*(I*e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)
- I*e^(-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*cos(-1/2*(b^2 - 4*a*c)/c)
 - c*(e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + e^(-1/2*(4*I*c^2*x^2 +
4*I*b*c*x + I*b^2)/c))*sin(-1/2*(b^2 - 4*a*c)/c))*sqrt((4*c^2*x^2 + 4*b*c*x
 + b^2)/abs(c)) + ((sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x +
 I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x +
 I*b^2)/c)) - 1))*b^2*cos(-1/2*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(erf(sqrt(1/
2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(sqrt(1
/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/2*(b^2 - 4
*a*c)/c) + (2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c)) - 1))*b*c*cos(-1/2*(b^2 - 4*a*c)/c) + (-2*I*sqrt(pi)*(erf(sqrt(1/2)*
```

```
sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(1/
2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/2*(b^2 - 4*
a*c)/c))*x)*cos(1/2*arctan2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + ((-I*sqr
t(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sq
rt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2
*cos(-1/2*(b^2 - 4*a*c)/c) - (sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/2*(b^2 - 4*a*c)/c) + ((-2*I*sqrt(pi)
*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(
pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*co
s(-1/2*(b^2 - 4*a*c)/c) - 2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/2*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan
2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)))*d*e/(c^2*sqrt((4*c^2*x^2 + 4*b*c*x
+ b^2)/abs(c))) + 1/192*sqrt(2)*(2*sqrt(2)*(8*c^3*x^3*abs(c) - b*c*(3*I*e^(
1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 3*I*e^(-1/2*(4*I*c^2*x^2 + 4*I*b
*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) + 3*b*c*(e^(1/2*(4*I*c^2
*x^2 + 4*I*b*c*x + I*b^2)/c) + e^(-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)
)*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))*((4*c^2*x^2 + 4*b*c*x + b^2)/abs(c))^(3
/2) + (6*b^3*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3
/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c
)/c) - b^3*(6*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 6*I*g
amma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 -
 4*a*c)/c) + (48*c^3*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) +
 gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2
 - 4*a*c)/c) - c^3*(48*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c
) - 48*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-
1/2*(b^2 - 4*a*c)/c))*x^3 + (72*b*c^2*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs
(c)*cos(-1/2*(b^2 - 4*a*c)/c) - b*c^2*(72*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4
*I*b*c*x + I*b^2)/c) - 72*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^
2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))*x^2 + (36*b^2*c*(gamma(3/2, 1/2*(4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c
*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) - b^2*c*(36*I*gamma(3/2, 1
/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 36*I*gamma(3/2, -1/2*(4*I*c^2*x^2
 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))*x)*cos(3/2*arct
an2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) - (3*(sqrt(pi)*(erf(sqrt(1/2)*sqrt
((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(
-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(-1/2*(b^2 - 4*a*c)/c)
+ (-3*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))
- 1) + 3*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/
c)) - 1))*b^5*sin(-1/2*(b^2 - 4*a*c)/c) + (24*(sqrt(pi)*(erf(sqrt(1/2)*sqrt
((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(
-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/2*(b^2 - 4*a*c)
/c) + (-24*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)
```

/c)) - 1) + 24*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I
*b^2)/c)) - 1))*b^2*c^3*sin(-1/2*(b^2 - 4*a*c)/c))*x^3 + (36*(sqrt(pi)*(erf
(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(
sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/
2*(b^2 - 4*a*c)/c) + (-36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I
*b*c*x + I*b^2)/c)) - 1) + 36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*sin(-1/2*(b^2 - 4*a*c)/c))*x^2 + (18
(sqrt(pi)(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) +
 sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*
b^4*c*cos(-1/2*(b^2 - 4*a*c)/c) + (-18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*
c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 18*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-
(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/2*(b^2 - 4*a*c)/c)
)*x)*cos(1/2*arctan2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) - (b^3*(6*I*gamma
(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 6*I*gamma(3/2, -1/2*(4*I*c
^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) + 6*b^3*(g
amma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c
^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c) + (c^3*(48
*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 48*I*gamma(3/2, -1
/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) +
 48*c^3*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -
1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))
*x^3 + (b*c^2*(72*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) - 7
2*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*cos(-1/2*(
b^2 - 4*a*c)/c) + 72*b*c^2*(gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2
)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*abs(c)*sin(-1/
2*(b^2 - 4*a*c)/c))*x^2 + (b^2*c*(36*I*gamma(3/2, 1/2*(4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c) - 36*I*gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)
)*abs(c)*cos(-1/2*(b^2 - 4*a*c)/c) + 36*b^2*c*(gamma(3/2, 1/2*(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c) + gamma(3/2, -1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)
/c))*abs(c)*sin(-1/2*(b^2 - 4*a*c)/c))*x)*sin(3/2*arctan2(2*(4*c^2*x^2 + 4*
b*c*x + b^2)/c, 0)) - ((-3*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1) + 3*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2
+ 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*cos(-1/2*(b^2 - 4*a*c)/c) - 3*(sqrt(pi)*
(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(
erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^5*sin(-1/
2*(b^2 - 4*a*c)/c) + ((-24*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c)) - 1) + 24*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2
 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c^3*cos(-1/2*(b^2 - 4*a*c)/c) - 24*(sqr
t(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + sqrt
(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c
^3*sin(-1/2*(b^2 - 4*a*c)/c))*x^3 + ((-36*I*sqrt(pi)*(erf(sqrt(1/2)*sqrt((4
*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 36*I*sqrt(pi)*(erf(sqrt(1/2)*sqr
t(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^3*c^2*cos(-1/2*(b^2 - 4*a*
c)/c) - 36*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/
c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/

```
c)) - 1))*b^3*c^2*sin(-1/2*(b^2 - 4*a*c)/c))*x^2 + ((-18*I*sqrt(pi)*(erf(sq
rt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 18*I*sqrt(pi)*(er
f(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^4*c*cos(-1/
2*(b^2 - 4*a*c)/c) - 18*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*
c*x + I*b^2)/c)) - 1))*b^4*c*sin(-1/2*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan2(
2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)))*e^2/(c^3*((4*c^2*x^2 + 4*b*c*x + b^2)
/abs(c))^(3/2)*abs(c))
```

Fricas [A]   time = 1.71046, size = 608, normalized size = 2.09

$$8\,c^3e^2x^3 + 24\,c^3dex^2 + 24\,c^3d^2x - 6\left(2\,c^2e^2x + 4\,c^2de - bce^2\right)\cos\left(cx^2 + bx + a\right)\sin\left(cx^2 + bx + a\right) + 3\left(\pi ce^2\sin\left(-\frac{b^2-4\,ac}{2\,c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*sin(c*x^2+b*x+a)^2,x, algorithm="fricas")

[Out] 1/48*(8*c^3*e^2*x^3 + 24*c^3*d*e*x^2 + 24*c^3*d^2*x - 6*(2*c^2*e^2*x + 4*c^
2*d*e - b*c*e^2)*cos(c*x^2 + b*x + a)*sin(c*x^2 + b*x + a) + 3*(pi*c*e^2*si
n(-1/2*(b^2 - 4*a*c)/c) - pi*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*cos(-1/2*(b^
2 - 4*a*c)/c))*sqrt(c/pi)*fresnel_cos((2*c*x + b)*sqrt(c/pi)/c) + 3*(pi*c*e
^2*cos(-1/2*(b^2 - 4*a*c)/c) + pi*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sin(-1/
2*(b^2 - 4*a*c)/c))*sqrt(c/pi)*fresnel_sin((2*c*x + b)*sqrt(c/pi)/c))/c^3

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^2 \sin^2\left(a + bx + cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*sin(c*x**2+b*x+a)**2,x)

[Out] Integral((d + e*x)**2*sin(a + b*x + c*x**2)**2, x)

Giac [C]   time = 1.34886, size = 726, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*sin(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3*e^2 + 1/2*d*x^2*e + 1/2*d^2*x + 1/8*sqrt(pi)*d^2*erf(-1/2*sqrt(c)*(
2*x + b/c)*(-I*c/abs(c) + 1))*e^(-1/2*(I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(-I*c/a
bs(c) + 1)) + 1/8*sqrt(pi)*d^2*erf(-1/2*sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1
))*e^(-1/2*(-I*b^2 + 4*I*a*c)/c)/(sqrt(c)*(I*c/abs(c) + 1)) - 1/8*(sqrt(pi)
*b*d*erf(-1/2*sqrt(c)*(2*x + b/c)*(-I*c/abs(c) + 1))*e^(-1/2*(I*b^2 - 4*I*a
*c - 2*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) - I*d*e^(2*I*c*x^2 + 2*I*b*x + 2*I
*a + 1))/c - 1/8*(sqrt(pi)*b*d*erf(-1/2*sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1
))*e^(-1/2*(-I*b^2 + 4*I*a*c - 2*c)/c)/(sqrt(c)*(I*c/abs(c) + 1)) + I*d*e^(
-2*I*c*x^2 - 2*I*b*x - 2*I*a + 1))/c - 1/32*((c*(-2*I*x - I*b/c) + 2*I*b)*e
^(2*I*c*x^2 + 2*I*b*x + 2*I*a + 2) - sqrt(pi)*(b^2 + I*c)*erf(-1/2*sqrt(c)*
(2*x + b/c)*(-I*c/abs(c) + 1))*e^(-1/2*(I*b^2 - 4*I*a*c - 4*c)/c)/(sqrt(c)*
(-I*c/abs(c) + 1)))/c^2 - 1/32*((c*(2*I*x + I*b/c) - 2*I*b)*e^(-2*I*c*x^2 -
 2*I*b*x - 2*I*a + 2) - sqrt(pi)*(b^2 - I*c)*erf(-1/2*sqrt(c)*(2*x + b/c)*(
I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 + 4*I*a*c - 4*c)/c)/(sqrt(c)*(I*c/abs(c) +
 1)))/c^2
```

3.34 $\int (d + ex) \sin^2 \left(a + bx + cx^2 \right) dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{\pi} \cos\left(2a - \frac{b^2}{2c}\right)(2cd - be)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{8c^{3/2}} + \frac{\sqrt{\pi} \sin\left(2a - \frac{b^2}{2c}\right)(2cd - be)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} - \frac{e \sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{(d+}{4}$$

```
[Out] (d + e*x)^2/(4*e) - ((2*c*d - b*e)*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(
b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])])/(8*c^(3/2)) + ((2*c*d - b*e)*Sqrt[Pi]*Fresn
elS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(8*c^(3/2)) - (e*
Sin[2*a + 2*b*x + 2*c*x^2])/(8*c)
```

Rubi [A]   time = 0.0933062, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3467, 3462, 3448, 3352, 3351}

$$-\frac{\sqrt{\pi} \cos\left(2a - \frac{b^2}{2c}\right)(2cd - be)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{8c^{3/2}} + \frac{\sqrt{\pi} \sin\left(2a - \frac{b^2}{2c}\right)(2cd - be)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} - \frac{e \sin\left(2a + 2bx + 2cx^2\right)}{8c} + \frac{(d+}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (d + e*x)^2/(4*e) - ((2*c*d - b*e)*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(
b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])])/(8*c^(3/2)) + ((2*c*d - b*e)*Sqrt[Pi]*Fresn
elS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^2/(2*c)])/(8*c^(3/2)) - (e*
Sin[2*a + 2*b*x + 2*c*x^2])/(8*c)
```

Rule 3467

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_),
 x_Symbol] :> Int[ExpandTrigReduce[(d + e*x)^m, Sin[a + b*x + c*x^2]^n, x],
 x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 1]
```

Rule 3462

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]*((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(e*Sin[a + b*x + c*x^2])/(2*c), x] + Dist[(2*c*d - b*e)/(2*c), Int[
Cos[a + b*x + c*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*
e, 0]
```

Rule 3448

```
Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\int (d + ex) \sin^2\left(a + bx + cx^2\right) dx = \int \left(\frac{1}{2}(d + ex) - \frac{1}{2}(d + ex)\cos\left(2a + 2bx + 2cx^2\right)\right) dx$$

$$= \frac{(d + ex)^2}{4e} - \frac{1}{2}\int (d + ex)\cos\left(2a + 2bx + 2cx^2\right) dx$$

$$= \frac{(d + ex)^2}{4e} - \frac{e\sin\left(2a + 2bx + 2cx^2\right)}{8c} - \frac{(4cd - 2be)\int \cos\left(2a + 2bx + 2cx^2\right) dx}{8c}$$

$$= \frac{(d + ex)^2}{4e} - \frac{e\sin\left(2a + 2bx + 2cx^2\right)}{8c} - \frac{\left((2cd - be)\cos\left(2a - \frac{b^2}{2c}\right)\right)\int \cos\left(\frac{(2b + 4cx)^2}{8c}\right) d}{4c}$$

$$= \frac{(d + ex)^2}{4e} - \frac{(2cd - be)\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right) C\left(\frac{b + 2cx}{\sqrt{c}\sqrt{\pi}}\right)}{8c^{3/2}} + \frac{(2cd - be)\sqrt{\pi} S\left(\frac{b + 2cx}{\sqrt{c}\sqrt{\pi}}\right)\sin\left(2a\right.}{8c^{3/2}}$$

Mathematica [A]   time = 0.437429, size = 140, normalized size = 0.93

$$\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)(-(2cd - be))\text{FresnelC}\left(\frac{b + 2cx}{\sqrt{\pi}\sqrt{c}}\right) + \sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)(2cd - be)S\left(\frac{b + 2cx}{\sqrt{c}\sqrt{\pi}}\right) + \sqrt{c}(2cx(2d + ex) - e\sin(2(a +}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (-((2*c*d - b*e)*Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c
]*Sqrt[Pi])]) + (2*c*d - b*e)*Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[P
i])]*Sin[2*a - b^2/(2*c)] + Sqrt[c]*(2*c*x*(2*d + e*x) - e*Sin[2*(a + x*(b
+ c*x))]))/(8*c^(3/2))
```

Maple [A]   time = 0.016, size = 170, normalized size = 1.1

$$-\frac{e \sin\left(2\,cx^2 + 2\,bx + 2\,a\right)}{8\,c} + \frac{be\sqrt{\pi}}{8}\left(\cos\left(\frac{-4\,ca + b^2}{2\,c}\right)\text{FresnelC}\left(\frac{2\,cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{-4\,ca + b^2}{2\,c}\right)\text{FresnelS}\left(\frac{2\,cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*sin(c*x^2+b*x+a)^2,x)
```

```
[Out] -1/8*e*sin(2*c*x^2+2*b*x+2*a)/c+1/8*e*b/c^(3/2)*Pi^(1/2)*(cos(1/2*(-4*a*c+b
^2)/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*Fresnel
S((2*c*x+b)/c^(1/2)/Pi^(1/2)))-1/4*Pi^(1/2)/c^(1/2)*d*(cos(1/2*(-4*a*c+b^2)
/c)*FresnelC((2*c*x+b)/c^(1/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*FresnelS((
2*c*x+b)/c^(1/2)/Pi^(1/2)))+1/2*d*x+1/4*e*x^2
```

Maxima [C]   time = 2.67835, size = 1786, normalized size = 11.91

<div align="center">result too large to display</div>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(2)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) - (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)
) - sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(2*I*c)) - ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) + (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)
) + sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(-2*I*c)))*sqrt(abs(c)) - 16*x*abs(c))*d/abs(c) + 1/32*sqrt(2
```

```
)*(sqrt(2)*(4*c^2*x^2 + c*(I*e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) -
I*e^(-1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c))*cos(-1/2*(b^2 - 4*a*c)/c) -
 c*(e^(1/2*(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c) + e^(-1/2*(4*I*c^2*x^2 + 4*
I*b*c*x + I*b^2)/c))*sin(-1/2*(b^2 - 4*a*c)/c))*sqrt((4*c^2*x^2 + 4*b*c*x +
 b^2)/abs(c)) + ((sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I
*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I
*b^2)/c)) - 1))*b^2*cos(-1/2*(b^2 - 4*a*c)/c) + (-I*sqrt(pi)*(erf(sqrt(1/2)
*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt(pi)*(erf(sqrt(1/2
)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/2*(b^2 - 4*a
*c)/c) + (2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)
/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)
/c)) - 1))*b*c*cos(-1/2*(b^2 - 4*a*c)/c) + (-2*I*sqrt(pi)*(erf(sqrt(1/2)*sq
rt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi)*(erf(sqrt(1/2)
*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/2*(b^2 - 4*a*
c)/c))*x)*cos(1/2*arctan2(2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)) + ((-I*sqrt(
pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + I*sqrt
(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b^2*c
os(-1/2*(b^2 - 4*a*c)/c) - (sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I
*b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I
*b*c*x + I*b^2)/c)) - 1))*b^2*sin(-1/2*(b^2 - 4*a*c)/c) + ((-2*I*sqrt(pi)*(
erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1) + 2*I*sqrt(pi
)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*b*c*x + I*b^2)/c)) - 1))*b*c*cos(
-1/2*(b^2 - 4*a*c)/c) - 2*(sqrt(pi)*(erf(sqrt(1/2)*sqrt((4*I*c^2*x^2 + 4*I*
b*c*x + I*b^2)/c)) - 1) + sqrt(pi)*(erf(sqrt(1/2)*sqrt(-(4*I*c^2*x^2 + 4*I*
b*c*x + I*b^2)/c)) - 1))*b*c*sin(-1/2*(b^2 - 4*a*c)/c))*x)*sin(1/2*arctan2(
2*(4*c^2*x^2 + 4*b*c*x + b^2)/c, 0)))*e/(c^2*sqrt((4*c^2*x^2 + 4*b*c*x + b^
2)/abs(c))))
```

Fricas [A]   time = 1.53868, size = 369, normalized size = 2.46

$$
\frac{2\,c^2ex^2 - \pi(2\,cd - be)\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{2\,c}\right)\mathrm{C}\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right) + \pi(2\,cd - be)\sqrt{\frac{c}{\pi}}\,\mathrm{S}\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right)\sin\left(-\frac{b^2-4\,ac}{2\,c}\right) + 4\,c^2dx - 2\,ce\cos}{8\,c^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*c^2*e*x^2 - pi*(2*c*d - b*e)*sqrt(c/pi)*cos(-1/2*(b^2 - 4*a*c)/c)*fr
esnel_cos((2*c*x + b)*sqrt(c/pi)/c) + pi*(2*c*d - b*e)*sqrt(c/pi)*fresnel_s
in((2*c*x + b)*sqrt(c/pi)/c)*sin(-1/2*(b^2 - 4*a*c)/c) + 4*c^2*d*x - 2*c*e*
cos(c*x^2 + b*x + a)*sin(c*x^2 + b*x + a))/c^2
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int (d + ex) \sin^2 \left(a + bx + cx^2 \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x**2+b*x+a)**2,x)
```

```
[Out] Integral((d + e*x)*sin(a + b*x + c*x**2)**2, x)
```

Giac [C]   time = 1.24996, size = 410, normalized size = 2.73

$$\frac{1}{4} x^2 e + \frac{1}{2} dx + \frac{\sqrt{\pi} d \operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x + \frac{b}{c}\right)\left(-\frac{ic}{|c|} + 1\right)\right) e^{\left(-\frac{ib^2 - 4iac}{2c}\right)}}{8\sqrt{c}\left(-\frac{ic}{|c|} + 1\right)} + \frac{\sqrt{\pi} d \operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x + \frac{b}{c}\right)\left(\frac{ic}{|c|} + 1\right)\right) e^{\left(-\frac{-ib^2 + 4iac}{2c}\right)}}{8\sqrt{c}\left(\frac{ic}{|c|} + 1\right)} - \frac{\sqrt{\pi} b \operatorname{erf}}{$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*sin(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2*e + 1/2*d*x + 1/8*sqrt(pi)*d*erf(-1/2*sqrt(c)*(2*x + b/c)*(-I*c/abs
(c) + 1))*e^(-1/2*(I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) + 1/8*sq
rt(pi)*d*erf(-1/2*sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 + 4
*I*a*c)/c)/(sqrt(c)*(I*c/abs(c) + 1)) - 1/16*(sqrt(pi)*b*erf(-1/2*sqrt(c)*(
2*x + b/c)*(-I*c/abs(c) + 1))*e^(-1/2*(I*b^2 - 4*I*a*c - 2*c)/c)/(sqrt(c)*(
-I*c/abs(c) + 1)) - I*e^(2*I*c*x^2 + 2*I*b*x + 2*I*a + 1))/c - 1/16*(sqrt(p
i)*b*erf(-1/2*sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 + 4*I*a
*c - 2*c)/c)/(sqrt(c)*(I*c/abs(c) + 1)) + I*e^(-2*I*c*x^2 - 2*I*b*x - 2*I*a
 + 1))/c
```

3.35 $\quad \int \sin^2\left(a + bx + cx^2\right) dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{x}{2}$$

[Out] x/2 - (Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi]
)])/(4*Sqrt[c]) + (Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*
a - b^2/(2*c)])/(4*Sqrt[c])

Rubi [A]   time = 0.0484573, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3449, 3448, 3352, 3351}

$$-\frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x + c*x^2]^2,x]

[Out] x/2 - (Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi]
)])/(4*Sqrt[c]) + (Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*
a - b^2/(2*c)])/(4*Sqrt[c])

Rule 3449

Int[Sin[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^(n_), x_Symbol] :> Int[ExpandTri
gReduce[Sin[a + b*x + c*x^2]^n, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 1]

Rule 3448

Int[Cos[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[Cos[(b^2 - 4*
a*c)/(4*c)], Int[Cos[(b + 2*c*x)^2/(4*c)], x], x] + Dist[Sin[(b^2 - 4*a*c)/
(4*c)], Int[Sin[(b + 2*c*x)^2/(4*c)], x], x] /; FreeQ[{a, b, c}, x] && NeQ[
b^2 - 4*a*c, 0]

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$
\int \sin^2\left(a + bx + cx^2\right) dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos\left(2a + 2bx + 2cx^2\right)\right) dx
$$

$$
= \frac{x}{2} - \frac{1}{2}\int \cos\left(2a + 2bx + 2cx^2\right) dx
$$

$$
= \frac{x}{2} - \frac{1}{2}\cos\left(2a - \frac{b^2}{2c}\right)\int \cos\left(\frac{(2b + 4cx)^2}{8c}\right) dx + \frac{1}{2}\sin\left(2a - \frac{b^2}{2c}\right)\int \sin\left(\frac{(2b + 4cx)^2}{8c}\right) dx
$$

$$
= \frac{x}{2} - \frac{\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)C\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right)\sin\left(2a - \frac{b^2}{2c}\right)}{4\sqrt{c}}
$$

Mathematica [A]   time = 0.0728906, size = 97, normalized size = 0.97

$$
\frac{-\sqrt{\pi}\cos\left(2a - \frac{b^2}{2c}\right)\text{FresnelC}\left(\frac{b+2cx}{\sqrt{\pi}\sqrt{c}}\right) + \sqrt{\pi}\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{b+2cx}{\sqrt{c}\sqrt{\pi}}\right) + 2\sqrt{c}x}{4\sqrt{c}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x + c*x^2]^2,x]
```

```
[Out] (2*Sqrt[c]*x - Sqrt[Pi]*Cos[2*a - b^2/(2*c)]*FresnelC[(b + 2*c*x)/(Sqrt[c]*
Sqrt[Pi])] + Sqrt[Pi]*FresnelS[(b + 2*c*x)/(Sqrt[c]*Sqrt[Pi])]*Sin[2*a - b^
2/(2*c)])/(4*Sqrt[c])
```

Maple [A]   time = 0.013, size = 72, normalized size = 0.7

$$
\frac{x}{2} - \frac{\sqrt{\pi}}{4}\left(\cos\left(\frac{-4ca + b^2}{2c}\right)\text{FresnelC}\left(\frac{2cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right) + \sin\left(\frac{-4ca + b^2}{2c}\right)\text{FresnelS}\left(\frac{2cx + b}{\sqrt{\pi}}\frac{1}{\sqrt{c}}\right)\right)\frac{1}{\sqrt{c}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*x^2+b*x+a)^2,x)
```

```
[Out] 1/2*x-1/4*Pi^(1/2)/c^(1/2)*(cos(1/2*(-4*a*c+b^2)/c)*FresnelC((2*c*x+b)/c^(1
/2)/Pi^(1/2))+sin(1/2*(-4*a*c+b^2)/c)*FresnelS((2*c*x+b)/c^(1/2)/Pi^(1/2)))
```

Maxima [C]   time = 2.16694, size = 410, normalized size = 4.1

$$-\frac{\sqrt{2}\sqrt{\pi}\left(\left(\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)-i\sin\left(\frac{1}{4}\pi+\frac{1}{2}\arctan(0,c)\right)+i\sin\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*(sqrt(2)*sqrt(pi)*(((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) - I*sin(1/4*pi + 1/2*arctan2(0, c)) + I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) - (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) + sin(1/4*pi + 1/2*arctan2(0, c)
) - sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(2*I*c)) - ((cos(1/4*pi + 1/2*arctan2(0, c)) + cos(-1/4*pi +
1/2*arctan2(0, c)) + I*sin(1/4*pi + 1/2*arctan2(0, c)) - I*sin(-1/4*pi + 1/
2*arctan2(0, c)))*cos(-1/2*(b^2 - 4*a*c)/c) + (I*cos(1/4*pi + 1/2*arctan2(0
, c)) + I*cos(-1/4*pi + 1/2*arctan2(0, c)) - sin(1/4*pi + 1/2*arctan2(0, c)
) + sin(-1/4*pi + 1/2*arctan2(0, c)))*sin(-1/2*(b^2 - 4*a*c)/c))*erf((2*I*c
*x + I*b)/sqrt(-2*I*c)))*sqrt(abs(c)) - 16*x*abs(c))/abs(c)
```

Fricas [A]   time = 1.46734, size = 238, normalized size = 2.38

$$-\frac{\pi\sqrt{\frac{c}{\pi}}\cos\left(-\frac{b^2-4\,ac}{2\,c}\right)C\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right)-\pi\sqrt{\frac{c}{\pi}}S\left(\frac{(2\,cx+b)\sqrt{\frac{c}{\pi}}}{c}\right)\sin\left(-\frac{b^2-4\,ac}{2\,c}\right)-2\,cx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(pi*sqrt(c/pi)*cos(-1/2*(b^2 - 4*a*c)/c)*fresnel_cos((2*c*x + b)*sqrt(
c/pi)/c) - pi*sqrt(c/pi)*fresnel_sin((2*c*x + b)*sqrt(c/pi)/c)*sin(-1/2*(b^
2 - 4*a*c)/c) - 2*c*x)/c
```

Sympy [A]   time = 1.20047, size = 83, normalized size = 0.83

$$\frac{x}{2} - \frac{\sqrt{\pi}\left(-\sin\left(2a - \frac{b^2}{2c}\right)S\left(\frac{2b+4cx}{2\sqrt{\pi}\sqrt{c}}\right) + \cos\left(2a - \frac{b^2}{2c}\right)C\left(\frac{2b+4cx}{2\sqrt{\pi}\sqrt{c}}\right)\right)\sqrt{\frac{1}{c}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x**2+b*x+a)**2,x)
```

```
[Out] x/2 - sqrt(pi)*(-sin(2*a - b**2/(2*c))*fresnels((2*b + 4*c*x)/(2*sqrt(pi)*s
qrt(c))) + cos(2*a - b**2/(2*c))*fresnelc((2*b + 4*c*x)/(2*sqrt(pi)*sqrt(c)
)))*sqrt(1/c)/4
```

Giac [C]   time = 1.32185, size = 165, normalized size = 1.65

$$\frac{1}{2}x + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x + \frac{b}{c}\right)\left(-\frac{ic}{|c|} + 1\right)\right)e^{\left(-\frac{ib^2-4iac}{2c}\right)}}{8\sqrt{c}\left(-\frac{ic}{|c|} + 1\right)} + \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x + \frac{b}{c}\right)\left(\frac{ic}{|c|} + 1\right)\right)e^{\left(-\frac{-ib^2+4iac}{2c}\right)}}{8\sqrt{c}\left(\frac{ic}{|c|} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/8*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + b/c)*(-I*c/abs(c) + 1))*e^(-1/
2*(I*b^2 - 4*I*a*c)/c)/(sqrt(c)*(-I*c/abs(c) + 1)) + 1/8*sqrt(pi)*erf(-1/2*
sqrt(c)*(2*x + b/c)*(I*c/abs(c) + 1))*e^(-1/2*(-I*b^2 + 4*I*a*c)/c)/(sqrt(c
)*(I*c/abs(c) + 1))
```

3.36 $\quad \int \dfrac{\sin^2\left(a+bx+cx^2\right)}{d+ex}\, dx$

Optimal. Leaf size=43

$$\frac{\log(d+ex)}{2e} - \frac{1}{2}\text{Unintegrable}\left(\frac{\cos\left(2a+2bx+2cx^2\right)}{d+ex}, x\right)$$

```
[Out] Log[d + e*x]/(2*e) - Unintegrable[Cos[2*a + 2*b*x + 2*c*x^2]/(d + e*x), x]/
2
```

Rubi [A]   time = 0.037116, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sin^2\left(a+bx+cx^2\right)}{d+ex}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[Sin[a + b*x + c*x^2]^2/(d + e*x),x]
```

```
[Out] Log[d + e*x]/(2*e) - Defer[Int][Cos[2*a + 2*b*x + 2*c*x^2]/(d + e*x), x]/2
```

Rubi steps

$$\int \frac{\sin^2\left(a+bx+cx^2\right)}{d+ex}\, dx = \int\left(\frac{1}{2(d+ex)} - \frac{\cos\left(2a+2bx+2cx^2\right)}{2(d+ex)}\right) dx$$

$$= \frac{\log(d+ex)}{2e} - \frac{1}{2}\int \frac{\cos\left(2a+2bx+2cx^2\right)}{d+ex}\, dx$$

Mathematica [A]   time = 7.40887, size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a+bx+cx^2\right)}{d+ex}\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sin[a + b*x + c*x^2]^2/(d + e*x),x]
```

```
[Out] Integrate[Sin[a + b*x + c*x^2]^2/(d + e*x), x]
```

Maple [A]   time = 0.23, size = 0, normalized size = 0.

$$\int \frac{\left(\sin\left(cx^2 + bx + a\right)\right)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c*x^2+b*x+a)^2/(e*x+d),x)
```

```
[Out] int(sin(c*x^2+b*x+a)^2/(e*x+d),x)
```

Maxima [A]   time = 0., size = 0, normalized size = 0.

$$\frac{-\frac{1}{2}\,e\int \frac{\cos\left(2\,cx^2+2\,bx\right)\cos(2\,a)-\sin\left(2\,cx^2+2\,bx\right)\sin(2\,a)}{\left(\cos(2\,a)^2+\sin(2\,a)^2\right)ex+\left(\cos(2\,a)^2+\sin(2\,a)^2\right)d}\, dx - \frac{1}{2}\,e\int \frac{\cos\left(2\,cx^2+2\,bx+2\,a\right)}{ex+d}\, dx + \log\left(ex + d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*(2*e*integrate(-1/4*(cos(2*c*x^2 + 2*b*x)*cos(2*a) - sin(2*c*x^2 + 2*b*
x)*sin(2*a))/((cos(2*a)^2 + sin(2*a)^2)*e*x + (cos(2*a)^2 + sin(2*a)^2)*d),
 x) - 2*e*integrate(1/4*cos(2*c*x^2 + 2*b*x + 2*a)/(e*x + d), x) + log(e*x
+ d))/e
```

Fricas [A]   time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\cos\left(cx^2 + bx + a\right)^2 - 1}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(-(cos(c*x^2 + b*x + a)^2 - 1)/(e*x + d), x)
```

Sympy [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(a + bx + cx^2\right)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x**2+b*x+a)**2/(e*x+d),x)
```

```
[Out] Integral(sin(a + b*x + c*x**2)**2/(d + e*x), x)
```

Giac [A]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin\left(cx^2 + bx + a\right)^2}{ex + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(c*x^2+b*x+a)^2/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sin(c*x^2 + b*x + a)^2/(e*x + d), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1  (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2  (* ::Package:: *)
3
4  (* ::Subsection:: *)
5  (*GradeAntiderivative[result,optimal]*)
6
7
8  (* ::Text:: *)
9  (*If result and optimal are mathematical expressions, *)
10 (*          GradeAntiderivative[result,optimal] returns*)
11 (*  "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (*  "C" if result involves higher level functions than necessary*)
14 (*  "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (*  "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
22      If[LeafCount[result]<=2*LeafCount[optimal],
23        "A",
24      "B"],
25    "C"],
26  If[FreeQ[result,Integrate] && FreeQ[result,Int],
27    "C",
28  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
69      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70    If[Head[expn]===Integrate || Head[expn]===Int,
71      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72    9]]]]]]]]]]

73

74
75  ElementaryFunctionQ[func_] :=
76    MemberQ[{
77          Exp,Log,
78          Sin,Cos,Tan,Cot,Sec,Csc,
79          ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80          Sinh,Cosh,Tanh,Coth,Sech,Csch,
81          ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82  },func]

83

84
85  SpecialFunctionQ[func_] :=
86    MemberQ[{
87          Erf, Erfc, Erfi,
88          FresnelS, FresnelC,
89          ExpIntegralE, ExpIntegralEi, LogIntegral,
90          SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91          Gamma, LogGamma, PolyGamma,
92          Zeta, PolyLog, ProductLog,
93          EllipticF, EllipticE, EllipticPi
94  },func]

95

96
97  HypergeometricFunctionQ[func_] :=
98    MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

99

100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]
```

4.0.2   Maple grading function

```
1  # File: GradeAntiderivative.mpl
2  # Original version thanks to Albert Rich emailed on 03/21/2017
3
4  #Nasser 03/22/2017  Use Maple leaf count instead since buildin
5  #Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
6  #Nasser 03/24/2017  corrected the check for complex result
7  #Nasser 10/27/2017  check for leafsize and do not call ExpnType()
8  #                   if leaf size is "too large". Set at 500,000
9  #Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
10 #                   see problem 156, file Apostol_Problems
```

```
11
12  GradeAntiderivative := proc(result,optimal)
13  local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
        debug:=false;
14
15      leaf_count_result:=leafcount(result);
16      #do NOT call ExpnType() if leaf size is too large. Recursion problem
17      if leaf_count_result > 500000 then
18          return "B";
19      fi;
20
21      leaf_count_optimal:=leafcount(optimal);
22
23      ExpnType_result:=ExpnType(result);
24      ExpnType_optimal:=ExpnType(optimal);
25
26      if debug then
27          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
        ExpnType_optimal);
28      fi;
29
30  # If result and optimal are mathematical expressions,
31  #  GradeAntiderivative[result,optimal] returns
32  #   "F" if the result fails to integrate an expression that
33  #       is integrable
34  #   "C" if result involves higher level functions than necessary
35  #   "B" if result is more than twice the size of the optimal
36  #       antiderivative
37  #   "A" if result can be considered optimal
38
39    #This check below actually is not needed, since I only
40    #call this grading only for passed integrals. i.e. I check
41    #for "F" before calling this. But no harm of keeping it here.
42    #just in case.
43
44
45    if not type(result,freeof('int')) then
46        return "F";
47    end if;
48
49
50    if ExpnType_result<=ExpnType_optimal then
51        if debug then
52            print("ExpnType_result<=ExpnType_optimal");
53        fi;
54        if is_contains_complex(result) then
55            if is_contains_complex(optimal) then
```

```
56              if debug then
57                  print("both result and optimal complex");
58              fi;
59              #both result and optimal complex
60              if leaf_count_result<=2*leaf_count_optimal then
61                  return "A";
62              else
63                  return "B";
64              end if
65          else #result contains complex but optimal is not
66              if debug then
67                  print("result contains complex but optimal is not");
68              fi;
69              return "C";
70          end if
71      else # result do not contain complex
72          # this assumes optimal do not as well
73          if debug then
74              print("result do not contain complex, this assumes optimal do not
    as well");
75          fi;
76          if leaf_count_result<=2*leaf_count_optimal then
77              if debug then
78                  print("leaf_count_result<=2*leaf_count_optimal");
79              fi;
80              return "A";
81          else
82              if debug then
83                  print("leaf_count_result>2*leaf_count_optimal");
84              fi;
85              return "B";
86          end if
87      end if
88  else #ExpnType(result) > ExpnType(optimal)
89      if debug then
90          print("ExpnType(result) > ExpnType(optimal)");
91      fi;
92      return "C";
93  end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```
102  is_contains_complex:= proc(expression)
103    return (has(expression,I));
104  end proc:
105
106  # The following summarizes the type number assigned an expression
107  # based on the functions it involves
108  # 1 = rational function
109  # 2 = algebraic function
110  # 3 = elementary function
111  # 4 = special function
112  # 5 = hyperpergeometric function
113  # 6 = appell function
114  # 7 = rootsum function
115  # 8 = integrate function
116  # 9 = unknown function
117
118  ExpnType := proc(expn)
119    if type(expn,'atomic') then
120        1
121    elif type(expn,'list') then
122      apply(max,map(ExpnType,expn))
123    elif type(expn,'sqrt') then
124      if type(op(1,expn),'rational') then
125          1
126      else
127          max(2,ExpnType(op(1,expn)))
128      end if
129    elif type(expn,'`^`') then
130      if type(op(2,expn),'integer') then
131        ExpnType(op(1,expn))
132      elif type(op(2,expn),'rational') then
133        if type(op(1,expn),'rational') then
134            1
135        else
136            max(2,ExpnType(op(1,expn)))
137        end if
138      else
139          max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140      end if
141    elif type(expn,'`+`') or type(expn,'`*`') then
142      max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143    elif ElementaryFunctionQ(op(0,expn)) then
144      max(3,ExpnType(op(1,expn)))
145    elif SpecialFunctionQ(op(0,expn)) then
146      max(4,apply(max,map(ExpnType,[op(expn)])))
147    elif HypergeometricFunctionQ(op(0,expn)) then
148      max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
149     elif AppellFunctionQ(op(0,expn)) then
150        max(6,apply(max,map(ExpnType,[op(expn)])))
151     elif op(0,expn)='int' then
152        max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154     end if
155  end proc:
156
157
158  ElementaryFunctionQ := proc(func)
159     member(func,[
160           exp,log,ln,
161           sin,cos,tan,cot,sec,csc,
162           arcsin,arccos,arctan,arccot,arcsec,arccsc,
163           sinh,cosh,tanh,coth,sech,csch,
164           arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165  end proc:
166
167  SpecialFunctionQ := proc(func)
168     member(func,[
169           erf,erfc,erfi,
170           FresnelS,FresnelC,
171           Ei,Ei,Li,Si,Ci,Shi,Chi,
172           GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173           EllipticF,EllipticE,EllipticPi])
174  end proc:
175
176  HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178  end proc:
179
180  AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182  end proc:
183
184  # u is a sum or product.  rest(u) returns all but the
185  # first term or factor of u.
186  rest := proc(u) local v;
187     if nops(u)=2 then
188        op(2,u)
189     else
190        apply(op(0,u),op(2..nops(u),u))
191     end if
192  end proc:
193
194  #leafcount(u) returns the number of nodes in u.
195  #Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
196   leafcount := proc(u)
197       MmaTranslator[Mma][LeafCount](u);
198   end proc:
```

4.0.3   Sympy grading function

```
1    #Dec 24, 2019. Nasser M. Abbasi:
2    #              Port of original Maple grading function by
3    #              Albert Rich to use with Sympy/Python
4    #Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
5    #              added 'exp_polar'
6    from sympy import *
7
8    def leaf_count(expr):
9        #sympy do not have leaf count function. This is approximation
10       return round(1.7*count_ops(expr))
11
12   def is_sqrt(expr):
13       if isinstance(expr,Pow):
14           if expr.args[1] == Rational(1,2):
15               return True
16           else:
17               return False
18       else:
19           return False
20
21   def is_elementary_function(func):
22       return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23               asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24               asinh,acosh,atanh,acoth,asech,acsch
25           ]
26
27   def is_special_function(func):
28       return func in [ erf,erfc,erfi,
29               fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30               gamma,loggamma,digamma,zeta,polylog,LambertW,
31               elliptic_f,elliptic_e,elliptic_pi,exp_polar
32           ]
33
34   def is_hypergeometric_function(func):
35       return func in [hyper]
36
37   def is_appell_function(func):
38       return func in [appellf1]
39
40   def is_atom(expn):
41       try:
```

```python
            if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
                return True
            else:
                return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
85          return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86      elif is_appell_function(expn.func):
87          m1 = max(map(expnType, list(expn.args)))
88          return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89      elif isinstance(expn,RootSum):
90          m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
91          return max(7,m1)
92      elif str(expn).find("Integral") != -1:
93          m1 = max(map(expnType, list(expn.args)))
94          return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95      else:
96          return 9
97
98  #main function
99  def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
    well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"
```

4.0.4   SageMath grading function

```
1  #Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
2  #                  Albert Rich to use with Sagemath. This is used to
3  #                  grade Fricas, Giac and Maxima results.
4  #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5  #                  'arctan2','floor','abs','log_integral'
6
7  from sage.all import *
8  from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13          print ("Enter tree(expr), expr=",expr)
14          print ("expr.operator()=",expr.operator())
15          print ("expr.operands()=",expr.operands())
16          print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()]+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34                     #since this estimate of leaf count is bit lower than
35                     #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow:   #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
```

```python
48  def is_elementary_function(func):
49      debug = False
50
51      m = func.name() in ['exp','log','ln',
52              'sin','cos','tan','cot','sec','csc',
53              'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54              'sinh','cosh','tanh','coth','sech','csch',
55              'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56          'arctan2','floor','abs'
57          ]
58      if debug:
59          if m:
60              print ("func ", func , " is elementary_function")
61          else:
62              print ("func ", func , " is NOT elementary_function")
63
64
65      return m
66
67  def is_special_function(func):
68      debug = False
69
70      if debug: print ("type(func)=", type(func))
71
72      m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73              'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74      sinh_integral'
74              'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
75              'polylog','lambert_w','elliptic_f','elliptic_e',
76              'elliptic_pi','exp_integral_e','log_integral']
77
78      if debug:
79          print ("m=",m)
80          if m:
81              print ("func ", func ," is special_function")
82          else:
83              print ("func ", func ," is NOT special_function")
84
85
86      return m
87
88
89  def is_hypergeometric_function(func):
90      return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91  ']
91
92  def is_appell_function(func):
```

```
93        return func.name() in ['hypergeometric']   #[appellf1] can't find this in
          sagemath

94

95  def is_atom(expn):

96

97        #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
          sagemath-equivalent-to-atomic-type-in-maple/
98        try:
99            if expn.parent() is SR:
100               return expn.operator() is None
101           if expn.parent() in (ZZ, QQ, AA, QQbar):
102               return expn in expn.parent() # Should always return True
103           if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104               return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105           return False

106

107       except AttributeError as error:
108           return False

109

110

111  def expnType(expn):
112       debug=False

113

114       if debug:
115           print (">>>>>Enter expnType, expn=", expn)
116           print (">>>>>is_atom(expn)=", is_atom(expn))

117

118       if is_atom(expn):
119           return 1
120       elif type(expn)==list:   #isinstance(expn,list):
121           return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122       elif  is_sqrt(expn):
123           if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124               return 1
125           else:
126               return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
          args[0]))
127       elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
128           if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
129               return expnType(expn.operands()[0])   #expnType(expn.args[0])
130           elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
          Rational)
131               if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                   return 1
```

```
133              else:
134                  return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
         args[0]))
135          else:
136              return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
         [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137      elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
         isinstance(expn,Add) or isinstance(expn,Mul)
138          m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139          m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140          return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141      elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
         func)
142          return max(3,expnType(expn.operands()[0]))
143      elif is_special_function(expn.operator()): #is_special_function(expn.func)
144          m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
         expn.args)))
145          return max(4,m1)   #max(4,m1)
146      elif is_hypergeometric_function(expn.operator()): #
         is_hypergeometric_function(expn.func)
147          m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
         expn.args)))
148          return max(5,m1)    #max(5,m1)
149      elif is_appell_function(expn.operator()):
150          m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
         expn.args)))
151          return max(6,m1)    #max(6,m1)
152      elif str(expn).find("Integral") != -1: #this will never happen, since it
153                  #is checked before calling the grading function that is passed.
154                  #but kept it here.
155          m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
         expn.args)))
156          return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157      else:
158          return 9

160  #main function
161  def grade_antiderivative(result,optimal):
162      debug = False;
163
164      if debug: print ("Enter grade_antiderivative for sagemath")
165
166      leaf_count_result  = leaf_count(result)
167      leaf_count_optimal = leaf_count(optimal)
168
169      if debug: print ("leaf_count_result=", leaf_count_result, "
         leaf_count_optimal=",leaf_count_optimal)
```

```
170
171
172     expnType_result  = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
        expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
        well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"
```